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Predictive modeling of iPhone 7 charge rates using least squares curve fitting

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Predictive Modeling of iPhone 7 Charge Rates Using Least Squares Curve Fitting

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Contents

1. Introduction

1.1. **Background.** Cell phones have become increasingly relied upon as a means of communication and entertainment as society has moved into a digital age. There are many well-known brands that have the "latest and greatest" smartphones, with each brand typically providing their own variety of options. Companies have even had to create better products that can cater to this desire of having things done at a moments' notice. Even top name brands such as Apple and Samsung are known to push for longer battery lives and even offer various forms of charging options for their phones when they are purchased. This can be attributed to the fact that when a consumer is choosing a cell phone to buy, a product's battery life can greatly affect their decision.

While these companies have conducted extensive research into ways of keeping a battery operational for a continuous period, no information regarding the time it would take for a fully dead battery at 0% to charge back up to 100%, can be found. While one can buy products that offer faster charging options for newer phones, where estimated charge times for obtaining a certain percentage charge are becoming more available [6], it still fails to answer the question of how long a cell phone takes to charge from 0% to 100% under different charging conditions. Knowing this data could potentially be another selling point for companies that could set them apart from their competitors. If one knows that a phone will become fully charged at a much faster pace than another or even if they just need it to be at a certain percentage and want to know how long they are going to have to sit by a charger, this information could be beneficial when making that initial purchase.

In a mathematical framework, this is one of the first studies to be done using least squares curve fitting with data collected about percentage charge rates of cell phones. While the process of least squares curve fitting is consistently applied to fitting different data points, there are no models or functions that depict the data of how long it would take for a fully dead battery to completely charge. Moreover, determining if the data would have a linear, quadratic, cubic, etc. relationship is an interesting question to look at. Since the type of data to be received was unknown at the time, using least squares curve fitting was an appropriate application because of its adaptive behavior to work regardless of what the correlation between the data is.

1.2. **Objectives.** In this project, a mathematical model will be created that can be used to predict an iPhone 7's charge rate. The objectives are two-fold:

1. To determine the amount of time it takes the iPhone 7 to charge from 0% to 100% under different usage conditions using the charger that comes with the phone (standard charging method).
2. To construct a mathematical function that represents the charge rate of an iPhone 7.

1.3. **Goal of the Study.** The goal of the study is to analyze an iPhone 7 under four different charging circumstances and to take that collected data and create a predictive model using least squares curve-fitting.

2. Methodology

2.1. **Least Squares Curve Fitting.** Least squares is a standard technique used in mathematical and statistical modeling. Specifically, least squares is a process for finding a curve that best fits a set of data. This process is used to solve over-determined linear systems, or systems where there are more equations than unknowns, as these types of systems end up being inconsistent. The goal of curve fitting is to find a functional relationship that represents, or fits, one's data points the best. This functional relationship typically follows a standard type of function such as linear, quadratic, cubic, etc. The curve is supposed to provide an optimal approximation of the minimized values between the sum of squares of errors of the data points and the values of the approximated curve [3, 5].

2.2. **Least Squares Theoretical Explanation.** Our two variables are x, which represents the time spent charging, and y, which represents the percentage charged.

$$\overrightarrow{x} = [x_1 \dots x_n]^T \text{ and } \overrightarrow{y} = [y_1 \dots y_n]^T \tag{1}$$

We want a mathematical relationship between $\overrightarrow{x}$ and $\overrightarrow{y}$, such that $\overrightarrow{y} = f(\overrightarrow{x})$.

We will use least squares curve fitting with a polynomial function to find this relationship. An example of such a polynomial function is

$$y = a_0 + a_1x^1 + a_2x^2 + \ldots + a_mx^m \tag{2}$$

where $n > m$.

By substituting (1) into (2), we get

$$\begin{aligned} y_1 &= a_0 + a_1x_1 + a_2x_1^2 + \ldots + a_mx_1^m \\ y_2 &= a_0 + a_1x_2 + a_2x_2^2 + \ldots + a_mx_2^m \\ &\vdots \\ y_n &= a_0 + a_1x_n + a_2x_n^2 + \ldots + a_mx_n^m \end{aligned}$$

or

$$\overrightarrow{y} = A\overrightarrow{a} \tag{3}$$

$$A = \begin{bmatrix} 1 & x_1 & \ldots & x_1^m \\ 1 & x_2 & \ldots & x_2^m \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & \ldots & x_n^m \end{bmatrix}, \quad \overrightarrow{a} = \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_m \end{bmatrix}, \text{ and } \overrightarrow{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

Notice that A is a $n \times (m+1)$ matrix and $\overrightarrow{a}$ is a $(m+1) \times 1$ vector. Throughout the rest of the paper, A will represent the matrix defined by (3) and $\overrightarrow{a}$ will represent the vector defined by (3).

The main problem with this is that experimental data may not satisfy (3). Therefore, an error will exist which will represent the Euclidean Distance between the vectors $\overrightarrow{y}$ and $A\overrightarrow{a}$.

$$\text{error} = \|\overrightarrow{y} - A\overrightarrow{a}\|$$

To fix this, one must minimize the error by finding a vector $\overrightarrow{a_0} \in \mathbb{R}^{m+1}$ such that

$$\|\overrightarrow{y} - A\overrightarrow{a_0}\| < \|\overrightarrow{y} - A\overrightarrow{a}\| \tag{4}$$

for all $\overrightarrow{a} \in \mathbb{R}^{m+1}$, where $A\overrightarrow{a} \in$ col(A) (column space of A).

Theorem 1 (*Best Approximation Theorem*). *[5]*

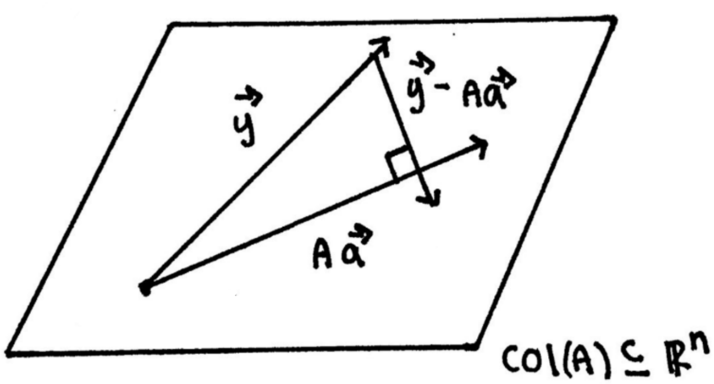
Let W be a finite dimensional subspace of an inner product space V. If $\overrightarrow{v} \in V$, then the projection of $\overrightarrow{u}$ onto W ($proj_W^{\overrightarrow{u}}$) is the best approximation to $\overrightarrow{u}$ from W in the sense that $\|\overrightarrow{u} - proj_W^{\overrightarrow{u}}\| \leq \|\overrightarrow{u} - W\|$ for all $\overrightarrow{w} \in W$ where $proj_{W^{\overrightarrow{u}}} \neq \overrightarrow{w}$.

By Theorem 1, (4) will be minimized if there exists a vector $\overrightarrow{a_0} \in \mathbb{R}^{m+1}$ such that A$\overrightarrow{a_0}$ is the projection of $\overrightarrow{y}$ onto col(A) relative to the Euclidean Inner Product. Thus $\overrightarrow{y} - A\overrightarrow{a_0}$ is orthogonal to col(A). That is,

$$(\overrightarrow{y} - A\overrightarrow{a_0}) \cdot A\overrightarrow{a} = 0 \tag{5}$$

for all $\overrightarrow{a} \in \mathbb{R}^{m+1}$ and $A\overrightarrow{a} \in$ col(A).

The geometric interpretation is below.



From (5),

$$(A\overrightarrow{a})^T(\overrightarrow{y} - A\overrightarrow{a_0}) = \overrightarrow{a}^T A^T(\overrightarrow{y} - A\overrightarrow{a_0}) = 0$$

$$\overrightarrow{a}^T(A^T\overrightarrow{y} - A^T A\overrightarrow{a_0}) = 0$$

$$(A^T\overrightarrow{y} - A^T A\overrightarrow{a_0}) \cdot \overrightarrow{a} = 0$$

Since the above statement could be true for all $\overrightarrow{a} \in \mathbb{R}^{m+1}$, the only way that the statement is true is if

$$A^T\overrightarrow{y} - A^T A\overrightarrow{a_0} = 0.$$

or

$$A^T A\overrightarrow{a_0} = A^T\overrightarrow{y} \tag{6}$$

If $A^T A$ is invertible, then $\overrightarrow{a_0} = (A^T A)^{-1} A^T\overrightarrow{y}$.

Theorem 2. *[5]*

Let A be an $n\times(m+1)$ matrix. If A has linearly independent columns, then $A^T A$ is invertible.

Proof. Let $\overrightarrow{x} \in N(A)$ (the null space of A). Then $A\overrightarrow{x} = 0$ by the definition of a null space. Then $A^T A\overrightarrow{x} = A^T * 0 = 0$. Thus $\overrightarrow{x} \in N(A^T A)$. Hence,

$$N(A) \subseteq N(A^T A). \tag{7}$$

Let $\overrightarrow{x} \in N(A^T A)$. Then $A^T A\overrightarrow{x} = 0$ implies $(A^T A\overrightarrow{x})^T\overrightarrow{x} = 0$ or $(A\overrightarrow{x})^T A\overrightarrow{x} = 0$. Then $\|A\overrightarrow{x}\| = 0$ or $A\overrightarrow{x} = 0$. Thus $\overrightarrow{x} \in N(A)$. Thus,

$$N(A^T A) \subseteq N(A). \tag{8}$$

Thus, by (7) and (8), $N(A^T A) = N(A)$. Since the nullities are equal, the ranks are also equal. Thus A and $A^T A$ have the same rank. Since $A^T A$ and A have the same rank and A has linearly independent columns, then $A^T A$ also has linearly independent columns. Since

A is an $n \times (m+1)$ matrix, A^T is an $(m+1) \times n$ matrix. Thus A^TA would be an $n \times n$, or a square matrix. Therefore, A^TA is invertible. □

Theorem 3. *[1]*

If at least two of the numbers $x_1 \dots x_n$ are distinct, then A has linearly independent columns.

Proof. A has linearly independent columns if no column is a nonzero linear combination of the other columns. Since all of the entries in Column 1 are the same, Column 1 is not a nonzero linear combination of Column 2 if $\exists\ i, j$ where $i \neq j$ such that $x_i \neq x_j$. But $x_i \neq x_j$ implies $x_i^k \neq x_j^k$, $1 \leq k \leq m$. Thus, no column of A is a nonzero combination of other columns if $\exists\ i, j$ where $i \neq j$ such that $x_i \neq x_j$. Therefore, A has linearly independent columns if at least two of the numbers $x_1 \dots x_n$ are distinct. □

3. Data Collection and Analysis

3.1. Description of Data.

Data Collection. The iPhone 7 was observed while charging under various charging situations. There was a total of four different charging situations analyzed with each being recorded ten times for a grand total of forty sets of data. The four charging situations include: charging without usage and without applications running in the background, charging without usage and with select applications running in the background, charging with usage and with applications running in the background, and charging while on airplane mode without applications running in the background. The applications that were chosen to be running in the background or to be used were ones that many individuals might have on their phone. These include Instagram, Snapchat, Messaging, Google, and YouTube. For the charging situation that involved using the phone, the same five applications as the ones mentioned above were used with each application being used for 30 minute time periods. The first application would be used for 30 minutes, the next application would be used for 30 minutes and so on until the phone reached a 100% charge. Doing this allowed for the usage to remain as consistent as possible over every charge period.

The percentage charge the phone had was recorded every two minutes from the time the phone was plugged in to whenever it reached a 100% charge. The phone was plugged in when it had 1% battery left so the percentage charge could be accurately recorded every two minutes. If the phone was plugged in when it was at 0%, there could not be an accurate record of its charge percentage as it would take a few minutes of charging for the screen to turn on so the percentage charge could be recorded. This would mean there would be inaccurate data for the first few minutes of charging. A phone that has only 1% battery left can be considered as practically dead for this study.

After collecting all of the data, entering them into Excel, and analyzing them and their graphs, an average of each time level was computed. This information was then made into their own graphs so the data would be easier to interpret. In MATLAB, the data of each charging situations average time at each two minute increment was entered and analyzed using the least squares process. The battery that was observed was the original battery that came with the phone when it was purchased. Similarly, the cable that was used to charge the devices was done with the cable that came with the phone at the time of purchase. Recording the phone's percentage charge was done the same way for every set time no matter what charging circumstance was being observed at the time.

3.2. **Description of Variables.** The two variables used in this study were the time the phone was charging for and the percentage charge the phone had. In this case the time spent charging was the independent variable and the percentage charge attained was the dependent variable.

3.3. **Excel Analysis.** All of the collected data was analyzed in Excel. The 40 sets of data were entered into four different spreadsheets based on the charging circumstance that was observed. After entering the data, an average of each charging circumstance was created. This synthesized the collected data into a simpler version that would be easier to understand. From there, three different types of graphs were created and interpreted. One of the graphs included the charging pattern of three sets of each charging circumstances' data in order to show any differences in the individual sets of data. This was repeated for each of the other three charging situations. The other graph that was looked at for each charging circumstance observed was the charge rates from around the 70 minute to the 90 minute charge times. This was where the shape of the graph was changing from a seemingly linear relationship to more of a curved relationship of some sort. It was interesting to look at this section of the data for that specific reason. The final graph depicted the curve of the average values of the charging situations. After creating the graphs, the graphs of the average values were analyzed for their line of best fit and their R^2 value. The R^2 value allows one to analyze how close or well a line of best fit correlates to the data. Both the graphs of the average values and information regarding their line of best fit and R^2 values are included below.

3.4. **MATLAB Analysis.** After analyzing the data in Excel, the data was then analyzed in MATLAB. As the data had already been averaged in Excel to create a representative set for each charging situation, each charging situation's average set was used in the MATLAB analysis. The average sets of data were entered into a .txt document to be used as a way of easily entering the data into MATLAB. After the data was pulled in, a vector, xdata, of one hundred data points representing each of the observed times was created. Additionally, another vector, y0, was created that used a specified column of data of one hundred points that represented the percentage charge at each of the points was pulled in as well. The type of equation one wanted to test or that one desired was typed out and the number of variables that needed to be calculated was also changed. There was some additional coding done to receive the least squares solution that can be seen in the Appendix section. When the code was run, the associated values of the coefficients in the least squares equation, the R^2 value, and the graph of the data with its associated fitted line were all calculated. The graph and set of coefficients for each charging situation are included below.

4. Results

4.1. **Data Results.** For all four of the charging circumstances analyzed, the data came out relatively similar. This was quite interesting considering the differences of each charging method. As previously stated, the four charging situations included: charging without usage and without applications running in the background, charging without usage and with select applications running in the background, charging with usage and with applications running in the background, and charging while on airplane mode without applications running in the background.

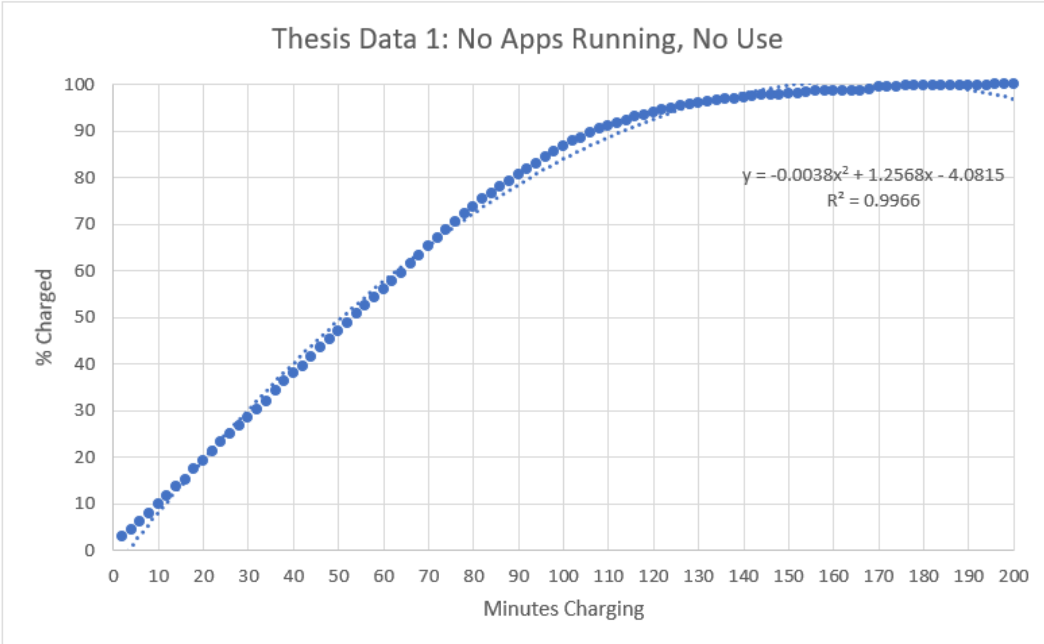
Looking at the data, it does not have consistent finishing times. Many of the 40 sets of data ended at different times meaning they all charged for differing amounts of time. For the charging circumstance where there was no use nor any applications running in the background, the ending charge times ranged from 138 minutes charging to 198 minutes charging with differing end times in between. For the charging circumstance where there was no use, but there were applications running in the background, end charging times were anywhere from 134 minutes charging to 174 minutes charging. When the phone was being used and had the five applications running in the background, the phone charged anywhere between 150 minutes to 204 minutes. While charging in airplane mode with no applications running in the background, the phone charging times ranged from 142 minutes to 186 minutes. The fact that there was no standard end time in each charging situation was quite interesting.

4.2. **Excel Analysis Results.**

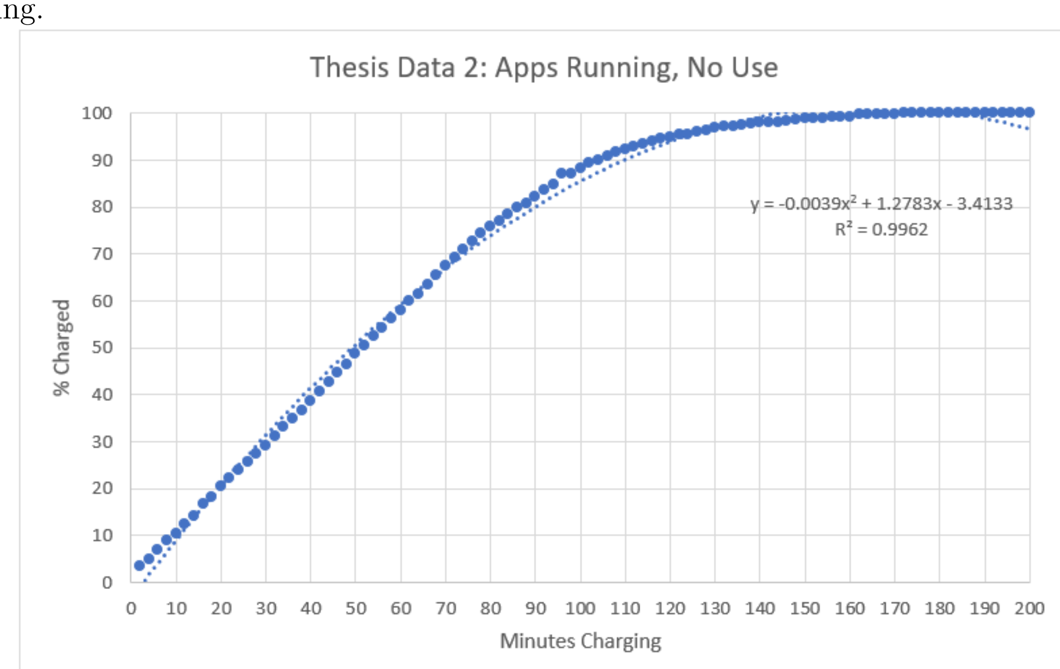
4.2.1. *Excel Graphs.* While many graphs were created and analyzed, the ones below are about the average of the 10 sets of data from each charging situation. For each graph, the bigger, bolder blue dots represent the data points and the smaller, lighter blue dots represent the line of best fit generated in Excel that corresponds to the equation included with each graph.

This first graph depicts the average of the first 10 sets of data where the iPhone 7 was charging with no applications running in the background nor any use of the device during charge time.

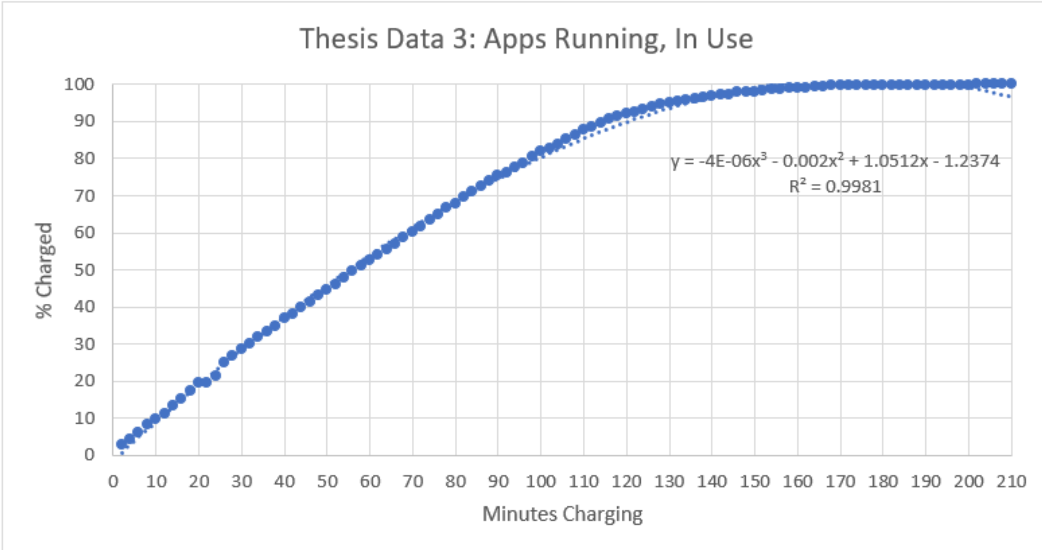
x



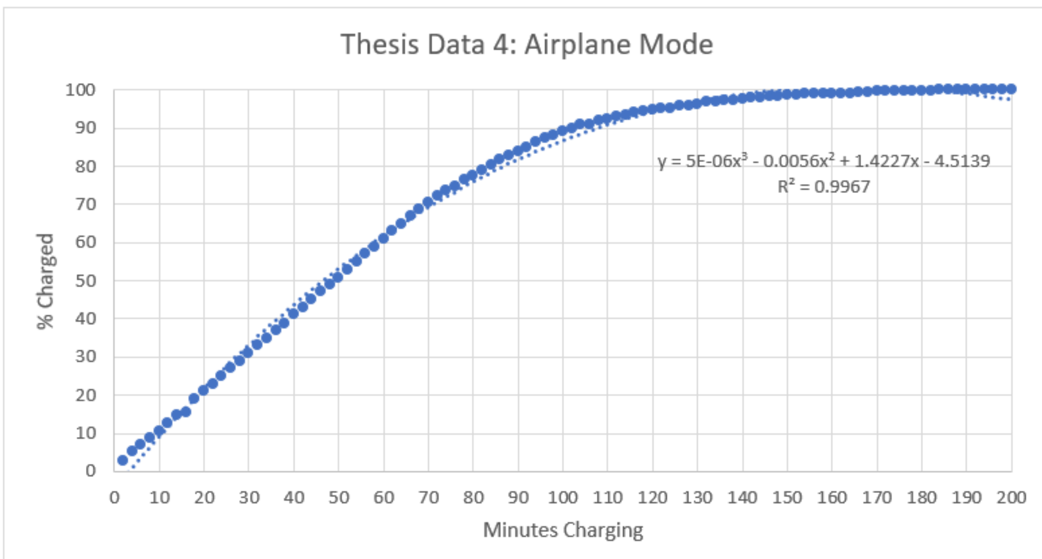
This second graph details the average of the second 10 sets of data where there were applications running in the background, but their was no use of the device while it was charging.



The third graph shows the average of the third 10 sets of data where there were applications running in the background and there was use of the device.



The final graph shows the average of the fourth 10 sets of data where the phone was in airplane mode while charging and there were no applications running in the background of the device.



4.2.2. *Discussion of Excel Results.* In general, the data started out as a linear relationship basically being a one-to-one correlation between the percentage charge and the amount of time spent charging. This pattern continued until the phone had been charging for around 70-80 minutes and had reached a 70-80% charge. At this point, the data began to level off with the phone taking roughly an additional 60-80+ minutes to charge 20% more to reach the 100% charge mark. The fact that the phone took double the amount of time to charge a fifth of 100% was quite compelling as this was not an expected outcome.

After analyzing the data and graphing it in Excel, the type of function of the line of best fit needed to be determined. To do this, one has to look at the shape the line of best fit has as well as the corresponding R^2 value. The R^2 value depicts how close the data is to the line of best fit or the variability between the predicted values and the observed values that is explained by the line. One wants this value to be as close to one as possible. If the R^2 value is equal to one, this would mean that the line of best fit fits the data perfectly and that all variability is explained.

While a linear relationship seemed like a good fit for the first half of the data, looking at the whole set of data, deemed the linear function to be an inefficient line of best fit. It did not graphically fit the data and the R^2 value was on the lower side. A logarithmic function was also tested with the data, but the R^2 value was too low to be an efficient line of best fit and the shape of the line did not fit well with the graphed data. Therefore, based on the shape of the graphs, there is one graph that has a negative cubic function as their line of best fit, one graph that has a positive cubic function as their line of best fit, and there are two graphs that have a negative quadratic function as the line of best.

The equations associated with the graphs in the Excel analysis are also worth mentioning as these equations represent the equation for the line of best fit and accompany the R^2 value. For the first graph depicting the first set of data, the equation is $y = 0.0038x^2 + 1.2568x - 4.0815$. The equation for second graph detailing the second set of data is $y = -0.0039x^2 + 1.2783x - 3.4133$. Both of these sets of data have lines of best fit that follow a negative quadratic equation. For the third graph with the third data set, the associated Excel equation is $y = -4e - 06x^3 - 0.002x^2 + 1.0512x - 1.2374$. Lastly, the equation from the fourth graph with the fourth set of data is $y = 5e - 06x^3 - 0.0056x^2 + 1.4227x - 4.5139$. While these last two sets of data have a line of best fit that follows a cubic function, it is interesting to note that one follows a negative cubic function while the other follows a positive cubic function. Specifically, the two charging situations where the phone was not in use while charging followed a negative quadratic function, while the charging situation where the phone was in use while charging followed a negative cubic function and where the phone was in airplane mode while charging followed a positive cubic function.

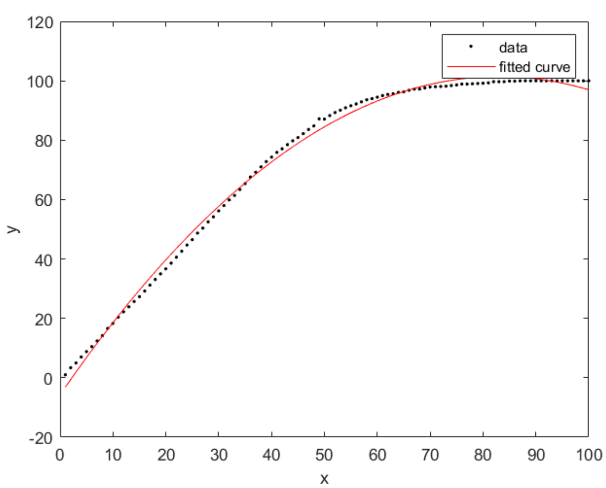
The R^2 values associated with each set of data is also important to mention. As seen on the graphs with a negative quadratic line of best fit, the R^2 value was around 0.996 for both graphs. Because our R^2 value is extremely close to one, it is evident that the negative quadratic function is the function that best fits the data. Similarly, the R^2 value for the graph depicting the data where the phone was in use with applications running in the background while charging was 0.9981. This R^2 value is also very close to one meaning it is a good fit for the data. For the final graph, the R^2 value is 0.9967. It is interesting to note that the R^2 value for the charging situation where the phone is in use while charging is higher than the other charging circumstances.

4.3. **MATLAB Analysis Results.**

4.3.1. *MATLAB Graphs.* Below are the graphs of each charging circumstances data in MATLAB. On each graph one can see the data line and the predicted line.

The first graph depicts the average data from the charging situation where there was no use of the device nor any applications running in the background while it was charging.

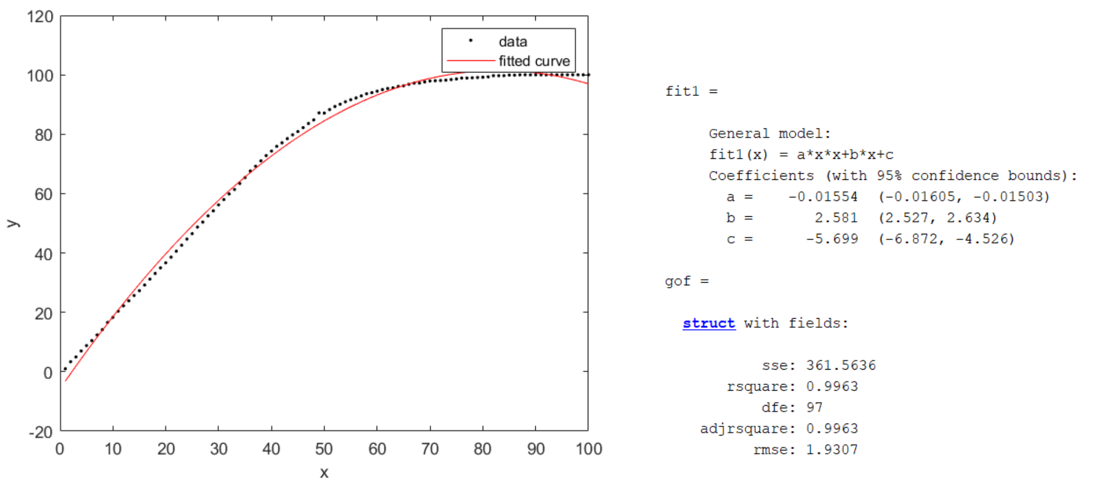
Thesis Data 1: No Apps, No Use




```
fit1 =

     General model:
     fit1(x) = a*x*x+b*x+c
     Coefficients (with 95% confidence bounds):
       a =    -0.01499  (-0.01549, -0.01449)
       b =       2.534  (2.482, 2.586)
       c =      -6.254  (-7.391, -5.116)

gof =

  struct with fields:

           sse: 340.0066
       rsquare: 0.9966
           dfe: 97
    adjrsquare: 0.9965
          rmse: 1.8722
```

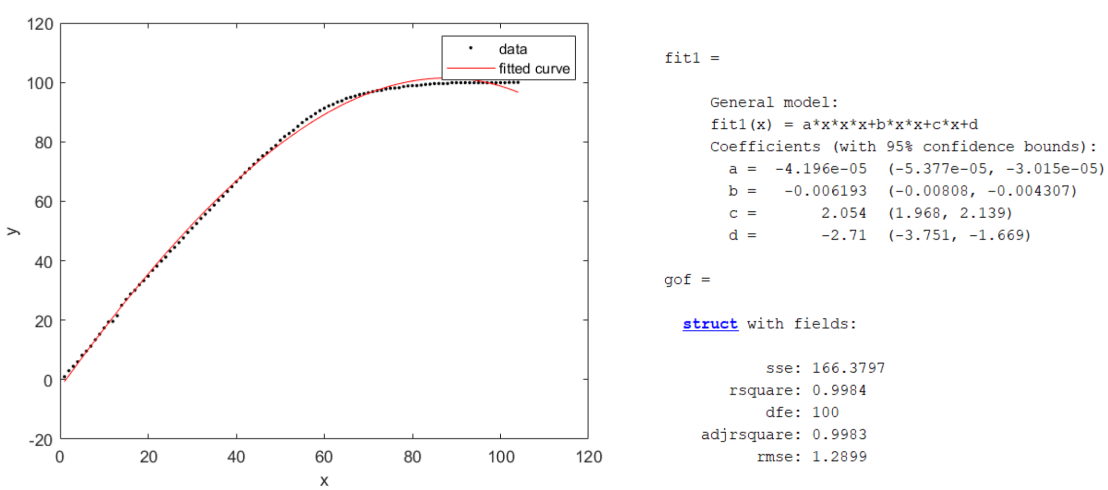
The second graph shows the average data of the charging situation where there was no use of the device, but there were applications running in the background during the time when the phone was charging.

Thesis Data 2: No Use, Apps Running



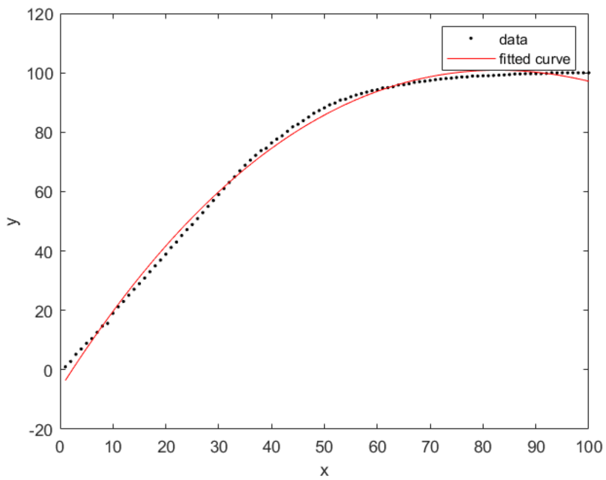
The third graph details the average charge time of the charging situation where the phone was in use with applications running in the background during its time charging.

Thesis Data 3: In Use, Apps Running



The fourth graph describes the charging situation where the phone was on airplane mode without applications running in the background when it was charging.

Thesis Data 4: Airplane Mode




```
fit1 =

     General model:
     fit1(x) = a*x*x*x+b*x*x+c*x+d
     Coefficients (with 95% confidence bounds):
       a =   3.177e-05  (1.263e-05, 5.091e-05)
       b =    -0.02091  (-0.02386, -0.01797)
       c =       2.809  (2.681, 2.937)
       d =      -6.379  (-7.881, -4.877)

gof =

  struct with fields:

           sse: 318.4316
       rsquare: 0.9967
           dfe: 96
    adjrsquare: 0.9966
          rmse: 1.8213
```

4.3.2. *Discussion of MATLAB Results.* Looking at the graphs that MATLAB produced, it is clear that the data starts off as a linear relationship before beginning to curve and then level off. The graphs look as though this leveling off starts around the 40 mark on the x-axis or the 80 minute mark of the charging time. This seems to be true for all four of the charging circumstances observed.

Based on the code used to create the graphs and to find the least squares solution, the created equations are important to document. These equations depict the line that minimize the error between the actual and the predicted values. As defined in the key on each graph, the black dots are the data of the average charge times, while the red dots depict the fitted curve or the line of best fit. For Thesis Data 1, the equation MATLAB created is $y = -0.01499x^2 + 2.534x - 6.254$. Looking at Thesis Data 2, $y = -0.01554x^2 + 2.581x - 5.699$ is the associated MATLAB equation. The next equation MATLAB came up with that is associated with Thesis Data 3 is $y = -4.196e - 05x^3 - 0.006193x^2 + 2.054x - 2.71$. For the final equation for Thesis Data 4, the equation from MATLAB is $y = 3.177e - 05x^3 -$

$0.02091x^2 + 2.809x - 6.379$. The first two sets of data both follow a negative quadratic line of best fit, while the third follows a negative cubic and the fourth follows a positive cubic line of best fit. It is interesting to see how the different sets of data ended up having different lines of best fit.

As previously explained, the R^2 value depicts how closely the data is to or how much variability is explained by the line of best fit. The R^2 value associated with the first set of data involving no use nor any running applications while charging is 0.9966. This value is very close to one, so one can assume that the line of best fit fits the data well. Similarly, for the second set of data where applications were running in the background, but there was no use of the device while it was charging, the R^2 value is 0.9963. This value is also very close to one and thus, the line of best fit can be deemed to fit the data well. For the third set of data involving the use of the phone and applications running while charging, the R^2 value calculated by MATLAB is 0.9984. Furthermore, for the final set of data where the phone was on airplane mode while charging, the associated R^2 value is 0.9967. These last two R^2 values for the third and fourth sets of data were also quite close to one, which signifies that they are suitable lines of best fit. MATLAB also provided other statistics that were not completely relevant for this study.

4.4. **Comparison of Excel and MATLAB Results.** Looking at the results from both Excel and MATLAB reveal quite a few intriguing things about their similarities and differences. First, as the data that was analyzed was the same for both systems, it makes sense that their graphs follow a similar shape; such as starting out linear and then leveling off. Moreover, it follows that since the data was the same, the lines of best fit would also follow a similar pattern or function type between the two systems. Something else that was similar was that the R^2 values for each charging circumstance were around the same value whether they were calculated in Excel or in MATLAB. Similarly, the numbers were all very close to one which implies that both systems can provide good lines of best fit for a set of data. Furthermore, as the lines of best fit followed similar function types, the values of the coefficients in the equations also fell quite close to one another.

While there were many similarities between the results of Excel and MATLAB, there were also a few differences. For starters, it seemed as though, in general, MATLAB was more specific with its values. It had more decimal places included in its equations and its R^2 values which leads one to believe it provides more accurate information regarding the data. Also, the R^2 value for Thesis Data 3 was higher, or a better fit, than the one provided in Excel. Following this, the analysis provided by MATLAB provided additional statistics such as the adjusted R^2 value, the sse, the rmse, among others. These provide useful information for other types of analysis and could be helpful for different projects in the future; even if they weren't needed for this one. Another difference worth noting is that the coefficients in the Excel and MATLAB equations do not match exactly. While they are close, they are not the same, which is an interesting thing to note. Since the same set of data was analyzed, it would have made sense to assume that the equations for the lines of best fit would have been the same. The fact that they are not is unexpected.

4.5. **Remarks.** There are many questions that come up when looking at the data collected. Some of these include: why are there varying end times for the same charging situation, does using the phone differently throughout the day affect how it charges later, why do different charging situations have similar stop times, and does the age of the battery affect the battery charge rate? In general, the age of the battery probably does have some impact on how quickly it would charge. Newer batteries aren't worn out and their overall quality is generally better. However, this can also vary phone by phone as some batteries might be flawed, even though they are unused, and be unusable. It is also rumored that Apple makes the system and battery lives of older phones start malfunctioning as newer phones get released [7]. This supposedly makes it so the newer phones have to be purchased because the older phones stop working. Phones have become necessities and to not have one is not an option for most people. While this is not something that has been proven nor has it happened to the device used in this study, it is worth mentioning as an older Apple product was observed.

One might attribute the varying end times for the same charging method to how the phone's battery is used throughout the day or even to other systems within the phone that can not be changed by its user. The fact that charging the phone under different circumstances did not result in differing end times nor differing lengths of charging periods, shows that there is not one specific charging circumstance that is better than others. This is contrary to what one might believe would happen if charged in different situations. However, since the data does not vastly differ between different charging circumstances, the model that is created to predict the charge time could possibly be generalized in later studies to work with many of the charging circumstances available. Similarly, in later studies, one could write a MATLAB code that would eliminate outliers in the data. This would make the data to be analyzed more specific and would allow different conclusions to be drawn.

Another interesting thing that was tested, was whether or not a phone charges faster when it is placed on airplane mode. It is a common belief that if one charges their phone while it is on airplane mode, it will charge faster because it is not using internet nor as much battery while it is being charged. This was not the case when looking at this set of data. Not only did it have inconsistent ending charge times, as the other sets of data did, but it did not charge the phone any faster than the other charging situations.

5. Appendix

5.1. **Device Disclaimer.** The iPhone 7 that was used in this study was purchased during the Summer of 2017. It contains all of its original parts, battery, etc. and has not experienced any noticeable problems in function or battery life. There have been no repairs or resets done on the device. While it is an older model of an iPhone, many individuals still have the device which makes it an acceptable source of study.

5.2. **Data Tables.** Below are the data tables for the four different charging situations. The chart depicting the data where there were applications running in the background and the phone was in use while charging (Table 3) is labeled with the application that was being used during that 30 minute period. While the messaging app was not used during the charging period, it remained running in the background for every trial. Using the messaging app during the charging period would have resulted in differing results because it would have been difficult to keep uniformity between the trials.

Thesis Data 1: No Apps, No Use											
	Trials:										
Time: (mins)	1	2	3	4	5	6	7	8	9	10	AVG
0	1	1	1	1	1	1	1	1	1	1	1
2	3	2	2	2	4	2	5	2	3	5	3
4	4	3	2	3	6	2	7	4	5	7	4.3
6	5	4	5	6	7	5	8	6	6	9	6.1
8	7	6	6	7	9	7	11	8	8	10	7.9
10	9	8	8	8	12	9	13	9	10	12	9.8
12	10	9	10	10	14	10	15	11	12	14	11.5
14	12	10	12	12	16	12	17	13	15	16	13.5
16	14	11	13	14	18	14	19	15	16	18	15.2
18	16	13	15	16	20	16	21	17	19	20	17.3
20	18	15	17	18	22	18	22	19	20	22	19.1

Continuation of Thesis Data 1											
22	20	18	19	20	23	20	24	21	23	23	21.1
24	23	19	21	23	25	22	26	23	25	26	23.3
26	24	21	23	24	27	23	28	24	27	28	24.9
28	26	23	24	26	29	25	29	26	29	29	26.6
30	28	25	26	28	30	27	30	28	31	31	28.4
32	30	26	28	30	32	28	32	30	32	33	30.1
34	32	28	30	32	34	30	33	32	34	35	32
36	34	30	32	35	36	32	36	34	36	37	34.2
38	36	32	34	37	38	34	38	36	38	39	36.2
40	37	34	36	39	40	35	40	37	40	41	37.9
42	39	36	37	40	42	37	41	39	42	43	39.6
44	41	38	39	42	44	39	43	41	44	45	41.6
46	43	40	42	44	46	41	45	43	45	47	43.6
48	45	41	43	46	48	42	47	45	47	48	45.2
50	47	43	45	48	50	43	49	46	49	50	47
52	49	45	47	49	52	45	51	47	51	52	48.8
54	51	48	49	51	54	47	53	48	53	54	50.8
56	52	49	50	53	56	48	55	50	55	56	52.4
58	54	51	52	55	58	51	56	51	57	58	54.3
60	56	53	54	57	60	52	58	53	58	59	56
62	58	55	56	59	61	54	59	55	60	61	57.8
64	60	57	58	60	63	56	61	57	61	63	59.6
66	62	59	60	62	65	58	63	59	63	65	61.6
68	63	61	62	64	66	60	65	60	65	67	63.3
70	65	63	63	66	69	61	67	62	67	69	65.2
72	67	65	65	68	71	63	68	63	69	70	66.9

Continuation of Thesis Data 1											
74	69	66	67	70	72	65	70	65	71	72	68.7
76	71	68	68	71	74	66	72	67	73	74	70.4
78	72	70	71	73	76	69	74	69	74	75	72.3
80	73	71	72	74	77	70	75	71	76	77	73.6
82	75	73	74	76	79	72	77	72	78	78	75.4
84	76	74	75	77	80	74	78	74	79	79	76.6
86	77	76	76	78	81	75	79	76	80	81	77.9
88	79	77	78	80	82	77	80	77	81	82	79.3
90	80	78	79	81	84	78	82	78	83	83	80.6
92	81	80	80	82	85	79	83	79	84	84	81.7
94	82	81	81	83	86	80	84	80	86	86	82.9
96	84	82	83	85	88	81	85	82	87	87	84.4
98	85	84	84	86	89	83	87	83	88	88	85.7
100	86	84	85	87	90	84	88	85	89	90	86.8
102	87	85	86	88	91	85	89	86	90	91	87.8
104	87	86	87	88	92	86	90	87	91	91	88.5
106	89	87	88	89	92	88	91	88	92	92	89.6
108	89	88	89	90	93	89	92	89	93	93	90.5
110	89	89	89	91	94	90	92	90	94	94	91.2
112	90	89	90	91	95	90	93	91	94	95	91.8
114	90	90	91	92	95	91	93	92	95	95	92.4
116	91	91	91	92	96	92	94	93	95	96	93.1
118	91	91	92	93	96	93	94	93	96	96	93.5
120	92	91	92	93	96	93	95	94	96	97	93.9
122	92	92	93	94	97	94	95	95	97	97	94.6
124	92	92	93	94	97	94	96	95	97	98	94.8

Continuation of Thesis Data 1											
126	93	93	93	94	98	95	96	96	98	98	95.4
128	93	93	94	95	98	95	97	96	98	98	95.7
130	93	93	94	95	98	96	97	96	98	99	95.9
132	94	94	94	95	99	96	97	97	99	99	96.4
134	94	94	95	95	99	96	97	97	99	99	96.5
136	94	94	95	96	99	97	98	97	99	100	96.9
138	94	94	95	96	99	97	98	98	99	100	97
140	95	94	95	96	100	97	98	98	100	100	97.3
142	95	95	96	96	100	98	98	98	100	100	97.6
144	95	95	96	97	100	98	98	98	100	100	97.7
146	95	95	96	97	100	98	99	99	100	100	97.9
148	95	95	96	97	100	98	99	99	100	100	97.9
150	96	95	96	97	100	98	99	99	100	100	98
152	96	95	97	97	100	99	99	99	100	100	98.2
154	96	96	97	97	100	99	99	99	100	100	98.3
156	96	96	97	98	100	99	99	100	100	100	98.5
158	96	96	97	98	100	99	100	100	100	100	98.6
160	96	96	97	98	100	99	100	100	100	100	98.6
162	97	96	97	98	100	99	100	100	100	100	98.7
164	97	96	97	98	100	99	100	100	100	100	98.7
166	97	96	97	98	100	99	100	100	100	100	98.7
168	97	96	98	98	100	100	100	100	100	100	98.9
170	97	100	98	100	100	100	100	100	100	100	99.5
172	97	100	98	100	100	100	100	100	100	100	99.5
174	97	100	98	100	100	100	100	100	100	100	99.5
176	97	100	100	100	100	100	100	100	100	100	99.7

Continuation of Thesis Data 1											
178	97	100	100	100	100	100	100	100	100	100	99.7
180	97	100	100	100	100	100	100	100	100	100	99.7
182	97	100	100	100	100	100	100	100	100	100	99.7
184	97	100	100	100	100	100	100	100	100	100	99.7
186	97	100	100	100	100	100	100	100	100	100	99.7
188	98	100	100	100	100	100	100	100	100	100	99.8
190	98	100	100	100	100	100	100	100	100	100	99.8
192	98	100	100	100	100	100	100	100	100	100	99.8
194	98	100	100	100	100	100	100	100	100	100	99.8
196	100	100	100	100	100	100	100	100	100	100	100
198	100	100	100	100	100	100	100	100	100	100	100
200	100	100	100	100	100	100	100	100	100	100	100

Thesis Data 2: No Use, Apps Running											
	Trials:										
Time: (mins)	1	2	3	4	5	6	7	8	9	10	AVG
0	1	1	1	1	1	1	1	1	1	1	1
2	5	1	3	4	2	3	4	5	2	5	3.4
4	6	3	5	6	3	4	6	7	3	7	5
6	8	5	7	7	5	6	9	9	5	9	7
8	11	7	9	10	6	7	11	11	6	10	8.8
10	14	8	11	10	8	9	12	13	8	12	10.5
12	15	10	12	12	10	11	15	15	10	14	12.4
14	17	12	15	14	11	13	16	16	12	16	14.2
16	20	13	17	17	14	15	19	18	14	19	16.6
18	21	16	19	19	15	17	20	20	16	20	18.3

Continuation of Thesis Data 2											
20	23	18	22	21	17	19	22	22	17	23	20.4
22	25	20	23	23	19	21	23	24	19	25	22.2
24	27	22	25	24	21	23	25	25	21	26	23.9
26	28	24	27	27	23	24	27	26	23	28	25.7
28	30	25	29	28	24	25	29	28	25	30	27.3
30	32	27	30	30	26	27	31	30	27	32	29.2
32	34	30	32	32	27	29	33	32	29	34	31.2
34	36	32	34	34	29	31	35	34	30	36	33.1
36	38	34	36	36	31	33	37	35	32	38	35
38	40	36	38	37	33	35	39	37	33	39	36.7
40	41	38	40	39	35	37	41	39	35	41	38.6
42	44	40	41	41	37	39	43	40	38	43	40.6
44	46	42	44	43	39	41	45	42	39	45	42.6
46	47	44	46	46	41	43	47	44	42	47	44.7
48	49	46	48	47	43	45	49	46	43	49	46.5
50	51	48	50	50	45	48	50	49	45	51	48.7
52	53	50	52	51	47	49	52	49	48	53	50.4
54	55	52	54	54	49	52	54	51	49	55	52.5
56	57	54	56	55	51	53	56	52	52	56	54.2
58	59	56	58	58	52	56	58	54	53	58	56.2
60	60	58	60	60	54	58	59	55	55	61	58
62	62	60	62	61	56	60	61	57	57	63	59.9
64	63	61	64	63	57	61	63	58	59	65	61.4
66	65	63	66	65	59	63	65	60	62	66	63.4
68	67	65	68	67	61	65	67	62	64	68	65.4
70	70	68	70	69	63	67	69	64	66	70	67.6

Continuation of Thesis Data 2											
72	72	70	71	71	65	68	71	66	67	71	69.2
74	73	72	73	72	67	71	73	67	69	73	71
76	75	74	75	74	68	72	74	70	71	75	72.8
78	77	75	76	75	70	74	76	71	73	76	74.3
80	78	76	77	77	73	75	77	74	74	78	75.9
82	80	78	78	78	73	76	78	75	76	79	77.1
84	81	79	80	80	75	78	79	76	77	80	78.5
86	82	80	81	81	77	79	81	78	78	81	79.8
88	83	82	82	82	78	80	82	79	79	82	80.9
90	85	83	83	83	79	81	83	80	81	84	82.2
92	86	84	85	85	80	83	85	81	82	85	83.6
94	87	85	86	86	82	84	86	82	84	86	84.8
96	89	87	87	87	93	86	87	84	85	87	87.2
98	90	88	88	88	84	86	88	85	86	88	87.1
100	91	89	89	89	86	88	89	86	87	89	88.3
102	92	90	90	90	87	88	90	88	88	90	89.3
104	92	91	90	91	88	89	91	89	89	91	90.1
106	93	92	91	91	89	90	91	90	90	92	90.9
108	94	92	92	92	90	91	92	90	91	92	91.6
110	94	93	92	92	91	91	93	91	92	93	92.2
112	95	94	93	93	91	92	93	92	92	94	92.9
114	96	94	93	94	92	93	94	93	93	94	93.6
116	96	95	94	94	92	93	94	93	94	95	94
118	97	95	94	94	93	94	95	94	94	95	94.5
120	97	96	95	95	94	94	95	94	95	95	95
122	98	96	95	95	94	94	96	95	95	96	95.4

Continuation of Thesis Data 2										
124	98	96	95	96	94	95	96	95	95	96	95.6
126	98	97	96	96	95	95	96	96	96	96	96.1
128	99	97	96	96	95	95	96	96	96	97	96.3
130	99	97	96	97	96	96	97	96	97	97	96.8
132	100	98	97	97	96	96	97	97	97	97	97.2
134	100	98	97	97	96	96	97	97	97	97	97.2
136	100	98	97	97	97	97	98	97	97	98	97.6
138	100	99	97	98	97	97	98	97	98	98	97.9
140	100	99	97	98	97	97	98	98	98	98	98
142	100	99	98	98	97	97	98	98	98	98	98.1
144	100	99	98	98	98	97	98	98	98	98	98.2
146	100	99	98	98	98	98	99	98	98	98	98.4
148	100	99	98	98	98	98	99	99	99	98	98.6
150	100	100	98	99	98	98	99	99	99	99	98.9
152	100	100	98	99	98	98	99	99	99	99	98.9
154	100	100	99	99	98	98	99	99	99	99	99
156	100	100	99	99	99	98	99	99	99	99	99.1
158	100	100	99	99	99	99	99	99	99	99	99.2
160	100	100	99	99	99	99	99	99	100	99	99.3
162	100	100	99	100	99	99	100	100	100	100	99.7
164	100	100	99	100	99	99	100	100	100	100	99.7
166	100	100	99	100	99	99	100	100	100	100	99.7
168	100	100	100	100	100	99	100	100	100	100	99.9
170	100	100	100	100	100	99	100	100	100	100	99.9
172	100	100	100	100	100	100	100	100	100	100	100
174	100	100	100	100	100	100	100	100	100	100	100

Continuation of Thesis Data 2											
176	100	100	100	100	100	100	100	100	100	100	100
178	100	100	100	100	100	100	100	100	100	100	100
180	100	100	100	100	100	100	100	100	100	100	100
182	100	100	100	100	100	100	100	100	100	100	100
184	100	100	100	100	100	100	100	100	100	100	100
186	100	100	100	100	100	100	100	100	100	100	100
188	100	100	100	100	100	100	100	100	100	100	100
190	100	100	100	100	100	100	100	100	100	100	100
192	100	100	100	100	100	100	100	100	100	100	100
194	100	100	100	100	100	100	100	100	100	100	100
196	100	100	100	100	100	100	100	100	100	100	100
198	100	100	100	100	100	100	100	100	100	100	100
200	100	100	100	100	100	100	100	100	100	100	100

Thesis Data 3: Apps Running, In Use										
	Trials:									
Time: (mins/App)	1	2	3	4	5	6	7	8	9	10	AVG
0 (No Use)	1	1	1	1	1	1	1	1	1	1	1
2 (No Use)	1	3	3	2	4	5	3	2	4	3	3
4 (No Use)	2	5	4	4	5	7	5	3	6	4	4.5
6 (No Use)	3	6	5	5	7	9	6	5	8	6	6
8 (No Use)	6	8	7	7	10	12	8	7	9	8	8.2
10 (No Use)	7	10	8	9	11	13	9	8	11	10	9.6
12 (No Use)	8	11	11	10	13	15	11	9	13	12	11.3
14 (No Use)	10	14	12	12	15	17	13	12	15	14	13.4
16 (No Use)	13	16	14	14	17	19	14	13	17	16	15.3

Continuation of Thesis Data 3										
18 (No Use)	14	18	16	16	19	21	17	16	19	18	17.4
20 (No Use)	16	20	18	18	21	23	19	18	21	20	19.4
22 (No Use)	19	22	20	20	22	25	21	2	23	22	19.6
24 (No Use)	21	24	22	23	24	27	23	2	25	24	21.5
26 (No Use)	23	25	23	25	26	28	24	23	27	26	25
28 (No Use)	25	28	25	27	27	30	26	25	29	28	27
30 (No Use)	26	30	27	28	29	32	28	27	31	30	28.8
32 (IG)	28	31	28	30	31	33	29	28	32	31	30.1
34 (IG)	30	31	30	31	33	35	32	30	34	33	31.9
36 (IG)	32	32	31	33	34	37	33	31	35	35	33.3
38 (IG)	33	33	33	34	36	39	34	33	37	36	34.8
40 (IG)	35	35	35	36	38	41	37	34	39	38	36.8
42 (IG)	36	36	36	38	40	42	39	35	41	39	38.2
44 (IG)	38	38	37	39	41	44	41	38	42	41	39.9
46 (IG)	39	39	39	40	43	45	43	39	44	41	41.2
48 (IG)	41	41	41	43	44	47	45	41	46	43	43.2
50 (IG)	42	43	42	44	46	48	46	42	47	45	44.5
52 (IG)	42	44	44	46	48	50	48	44	49	46	46.1
54 (IG)	43	46	46	48	50	51	49	45	51	48	47.7
56 (IG)	44	48	48	50	51	53	51	47	53	50	49.5
58 (IG)	46	49	49	52	53	55	52	49	54	51	51
60 (IG)	47	51	50	53	55	56	54	50	56	53	52.5
62 (Snap)	49	53	52	55	56	57	55	52	58	55	54.2
64 (Snap)	50	54	52	56	58	59	57	54	60	56	55.6
66 (Snap)	52	55	54	58	59	61	59	55	61	57	57.1
68 (Snap)	53	57	56	60	61	62	60	56	63	59	58.7

Continuation of Thesis Data 3											
70 (Snap)	55	58	57	61	62	64	62	58	65	61	60.3
72 (Snap)	56	60	59	63	64	65	63	59	66	62	61.7
74 (Snap)	58	61	60	64	65	67	65	61	68	64	63.3
76 (Snap)	60	62	62	66	67	69	66	63	69	65	64.9
78 (Snap)	61	64	64	68	69	70	68	64	71	67	66.6
80 (Snap)	63	65	65	69	70	72	69	66	73	68	68
82 (Snap)	64	67	67	70	72	74	71	67	74	70	69.6
84 (Snap)	66	68	68	72	73	75	72	69	76	71	71
86 (Snap)	67	69	70	73	75	76	74	70	78	73	72.5
88 (Snap)	68	71	71	74	77	78	75	72	79	74	73.9
90 (Snap)	70	72	73	76	78	79	76	73	80	76	75.3
92 (YT)	71	72	74	77	79	80	78	74	81	77	76.3
94 (YT)	73	74	75	79	80	81	79	76	82	78	77.7
96 (YT)	74	75	77	80	81	82	80	77	83	79	78.8
98 (YT)	76	76	78	81	83	84	82	79	85	81	80.5
100 (YT)	77	78	80	83	84	85	83	80	86	82	81.8
102 (YT)	78	79	81	84	85	86	84	81	87	83	82.8
104 (YT)	79	80	82	85	87	87	85	82	88	84	83.9
106 (YT)	81	81	83	86	88	88	86	84	89	86	85.2
108 (YT)	82	83	84	88	89	89	88	85	90	87	86.5
110 (YT)	83	84	86	89	90	90	89	86	91	88	87.6
112 (YT)	84	84	87	90	91	91	90	87	92	89	88.5
114 (YT)	86	85	88	90	92	92	91	88	93	90	89.5
116 (YT)	87	87	89	91	93	93	91	90	93	91	90.5
118 (YT)	88	88	90	92	93	94	92	91	94	91	91.3
120 (YT)	89	89	91	92	94	95	93	92	94	92	92.1

Continuation of Thesis Data 3										
122 (Google)	90	89	92	94	94	95	93	92	95	92	92.6
124 (Google)	91	90	93	94	95	96	94	93	95	93	93.4
126 (Google)	92	91	93	95	95	96	94	93	95	94	93.8
128 (Google)	93	92	94	95	96	97	95	94	96	94	94.6
130 (Google)	94	92	95	95	96	97	95	95	96	95	95
132 (Google)	94	93	95	96	97	97	96	95	96	95	95.4
134 (Google)	95	94	96	96	97	98	96	95	97	95	95.9
136 (Google)	95	94	96	97	97	98	96	96	97	96	96.2
138 (Google)	96	94	96	97	98	98	97	96	97	96	96.5
140 (Google)	96	95	97	97	98	99	97	97	97	96	96.9
142 (Google)	97	95	97	97	98	99	97	97	98	97	97.2
144 (Google)	97	95	97	98	98	99	97	97	98	97	97.3
146 (Google)	97	96	98	98	99	99	98	98	98	97	97.8
148 (Google)	98	96	98	98	99	100	98	98	98	97	98
150 (Google)	98	96	98	98	99	100	98	98	98	98	98.1
152 (IG)	98	97	98	98	99	100	98	98	98	98	98.2
154 (IG)	99	97	99	99	99	100	98	98	99	98	98.6
156 (IG)	99	97	99	99	100	100	98	99	99	98	98.8
158 (IG)	99	97	99	99	100	100	99	99	99	98	98.9
160 (IG)	99	97	99	99	100	100	99	99	99	98	98.9
162 (IG)	99	98	99	99	100	100	99	99	99	99	99.1
164 (IG)	100	98	100	99	100	100	99	99	99	99	99.3
166 (IG)	100	98	100	99	100	100	99	100	99	99	99.4
168 (IG)	100	98	100	100	100	100	99	100	100	99	99.6
170 (IG)	100	98	100	100	100	100	99	100	100	99	99.6
172 (IG)	100	98	100	100	100	100	99	100	100	99	99.6

Continuation of Thesis Data 3										
174 (IG)	100	98	100	100	100	100	99	100	100	99	99.6
176 (IG)	100	99	100	100	100	100	100	100	100	100	99.9
178 (IG)	100	99	100	100	100	100	100	100	100	100	99.9
180 (IG)	100	99	100	100	100	100	100	100	100	100	99.9
182 (Snap)	100	99	100	100	100	100	100	100	100	100	99.9
184 (Snap)	100	99	100	100	100	100	100	100	100	100	99.9
186 (Snap)	100	99	100	100	100	100	100	100	100	100	99.9
188 (Snap)	100	99	100	100	100	100	100	100	100	100	99.9
190 (Snap)	100	99	100	100	100	100	100	100	100	100	99.9
192 (Snap)	100	99	100	100	100	100	100	100	100	100	99.9
194 (Snap)	100	99	100	100	100	100	100	100	100	100	99.9
196 (Snap)	100	99	100	100	100	100	100	100	100	100	99.9
198 (Snap)	100	99	100	100	100	100	100	100	100	100	99.9
200 (Snap)	100	99	100	100	100	100	100	100	100	100	99.9
202 (Snap)	100	100	100	100	100	100	100	100	100	100	100
204 (Snap)	100	100	100	100	100	100	100	100	100	100	100
206 (Snap)	100	100	100	100	100	100	100	100	100	100	100
208 (Snap)	100	100	100	100	100	100	100	100	100	100	100
210 (Snap)	100	100	100	100	100	100	100	100	100	100	100

Thesis Data 4: Airplane Mode											
	Trials:										
Time: (mins)	1	2	3	4	5	6	7	8	9	10	AVG
0	1	1	1	1	1	1	1	1	1	1	1
2	5	2	1	2	2	3	4	3	5	1	2.8
4	7	5	6	4	4	4	5	5	7	5	5.2

Continuation of Thesis Data 4

6	9	7	7	6	4	6	8	7	9	7	7
8	11	9	9	7	7	8	9	9	10	10	8.9
10	13	10	10	9	8	10	11	11	12	11	10.5
12	15	12	12	11	11	12	13	13	14	13	12.6
14	17	14	14	14	12	14	16	15	16	15	14.7
16	20	16	16	16	1	16	18	17	19	17	15.6
18	22	18	18	18	17	18	20	19	21	19	19
20	23	20	21	21	20	20	22	21	23	21	21.2
22	25	22	23	22	21	22	24	23	25	23	23
24	27	24	25	24	24	24	26	25	27	25	25.1
26	30	25	27	27	25	26	28	27	29	27	27.1
28	31	28	29	28	27	28	30	29	31	29	29
30	33	30	31	30	29	30	32	30	33	31	30.9
32	35	32	33	33	31	32	34	32	35	33	33
34	38	34	34	35	32	35	36	34	37	35	35
36	40	36	37	37	34	36	38	36	38	37	36.9
38	42	37	39	39	36	39	40	38	40	39	38.9
40	44	40	41	41	39	41	43	40	43	40	41.2
42	46	42	43	43	41	43	44	42	44	42	43
44	48	44	45	46	43	45	47	44	46	44	45.2
46	50	47	47	47	46	47	48	46	48	46	47.2
48	51	48	49	49	47	48	51	48	50	48	48.9
50	54	50	51	51	49	50	52	50	52	50	50.9
52	55	52	53	53	51	52	55	52	54	52	52.9
54	57	54	55	56	53	54	57	54	56	54	55
56	59	56	57	58	55	56	59	56	58	56	57

Continuation of Thesis Data 4											
58	61	58	59	60	57	58	61	58	60	58	59
60	63	60	61	62	59	60	63	60	62	60	61
62	65	63	63	64	61	62	65	62	64	62	63.1
64	67	64	65	66	63	64	67	64	66	64	65
66	69	66	67	68	65	66	69	66	68	66	67
68	71	68	69	70	67	68	70	68	70	68	68.9
70	73	70	70	71	69	70	72	70	71	70	70.6
72	74	72	72	72	71	72	74	71	73	71	72.2
74	76	73	73	74	73	74	75	73	74	73	73.8
76	77	74	74	75	74	75	73	74	76	74	74.6
78	78	75	76	77	76	76	78	75	77	76	76.4
80	79	77	77	78	77	78	79	77	78	77	77.7
82	80	78	78	79	79	79	80	78	79	78	78.8
84	82	79	80	81	80	80	82	79	81	79	80.3
86	84	81	81	82	81	82	83	81	82	81	81.8
88	84	82	82	83	82	83	84	82	83	82	82.7
90	86	83	83	84	84	84	85	83	84	83	83.9
92	87	84	84	85	85	86	86	84	86	84	85.1
94	88	85	85	87	86	87	88	85	87	86	86.4
96	89	86	86	87	87	88	89	86	88	87	87.3
98	90	87	87	88	88	89	89	87	89	88	88.2
100	91	88	88	89	90	90	90	88	89	89	89.2
102	91	89	88	90	90	91	91	89	90	89	89.8
104	92	90	89	91	91	92	92	90	91	90	90.8
106	92	90	90	91	92	92	92	90	91	91	91.1
108	93	91	90	92	93	93	93	91	92	91	91.9

Continuation of Thesis Data 4

110	94	91	91	92	94	94	94	91	92	92	92.5
112	94	92	91	93	94	94	94	92	93	93	93
114	95	93	92	93	94	95	95	92	93	93	93.5
116	95	93	92	94	95	95	95	93	94	94	94
118	95	93	92	94	96	96	96	93	94	94	94.3
120	96	94	93	95	96	96	96	94	95	94	94.9
122	96	94	93	95	96	96	96	94	95	95	95
124	96	94	93	95	97	97	97	94	95	95	95.3
126	97	95	94	96	97	97	97	95	96	95	95.9
128	97	95	94	96	97	97	97	95	96	96	96
130	97	95	94	96	98	98	98	95	96	96	96.3
132	98	96	95	96	98	98	98	96	97	96	96.8
134	98	96	95	97	98	98	98	96	97	97	97
136	98	96	95	97	99	99	98	96	97	97	97.2
138	98	97	95	97	99	99	99	96	97	97	97.4
140	98	97	95	97	100	99	99	97	98	97	97.7
142	99	97	96	97	100	99	99	97	98	97	97.9
144	99	97	96	98	100	99	99	97	98	98	98.1
146	99	97	96	98	100	100	99	97	98	98	98.2
148	99	98	96	98	100	100	100	97	98	98	98.4
150	99	98	97	98	100	100	100	98	98	98	98.6
152	99	98	97	98	100	100	100	98	98	98	98.6
154	100	98	97	98	100	100	100	98	99	99	98.9
156	100	98	97	98	100	100	100	98	99	99	98.9
158	100	98	97	99	100	100	100	98	99	99	99
160	100	98	97	99	100	100	100	98	99	99	99

Continuation of Thesis Data 4											
162	100	99	97	99	100	100	100	98	99	99	99.1
164	100	99	97	99	100	100	100	99	99	99	99.2
166	100	99	98	99	100	100	100	99	99	99	99.3
168	100	99	98	99	100	100	100	99	100	99	99.4
170	100	99	98	100	100	100	100	99	100	100	99.6
172	100	99	98	100	100	100	100	99	100	100	99.6
174	100	99	98	100	100	100	100	100	100	100	99.7
176	100	99	98	100	100	100	100	100	100	100	99.7
178	100	99	98	100	100	100	100	100	100	100	99.7
180	100	99	98	100	100	100	100	100	100	100	99.7
182	100	100	98	100	100	100	100	100	100	100	99.8
184	100	100	100	100	100	100	100	100	100	100	100
186	100	100	100	100	100	100	100	100	100	100	100
188	100	100	100	100	100	100	100	100	100	100	100
190	100	100	100	100	100	100	100	100	100	100	100
192	100	100	100	100	100	100	100	100	100	100	100
194	100	100	100	100	100	100	100	100	100	100	100
196	100	100	100	100	100	100	100	100	100	100	100
198	100	100	100	100	100	100	100	100	100	100	100
200	100	100	100	100	100	100	100	100	100	100	100

5.3. **MATLAB Code.** Below is the code used to calculate the least squares solution for the averages of each of the four sets of data. It has been generalized for the sake of adding it to this paper.

```
clear; clc;

Data=load('C:/Users/Acer/Downloads/ThesisDat.txt')

%n= Case from ThesisDat.txt; (1-4)

xdata = (1:1:100)'; % you have 100 values

y0 = Data(1:100,n); % this is the average values for Case n

f = fittype('a*x*x*x+b*x*x+c*x+d'); % change based on function type is desired

[fit1,gof,fitinfo] = fit(xdata,y0,f,'StartPoint',y0(1:4))

plot(fit1,'r-',xdata,y0,'k.')
```

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