Estimating Load-Deflection Characteristics for the Shaft Resistance of Piles Using Hyperbolic Strain Softening

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Introduction

- The load-displacement characteristics of deep foundations is an important topic for their design and use
- The theory of annular deflection is an important component in our understanding of shaft resistance
- Soils are inherently nonlinear in behavior; the hyperbolic model is a reasonable one to predict that non-linearity

Objectives of the Research

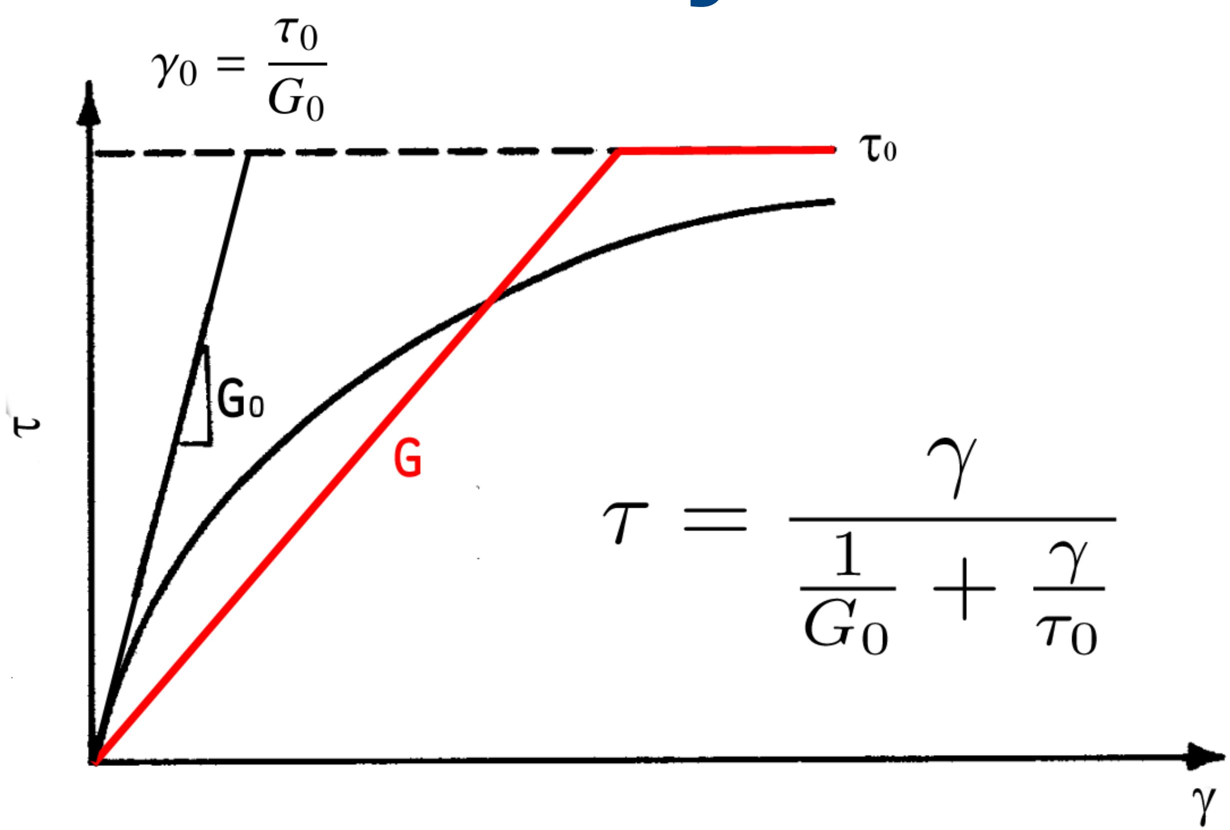
- To combine these two theories and use them to predict the deflection of a pile element in the region before the separation of the soil and the pile takes place
- To generalize the results sufficiently to make them applicable to a wide variety of soil and pile conditions (non-dimensionality)

Hyperbolic Stress-Strain Theory

Dimensionless Parameters:

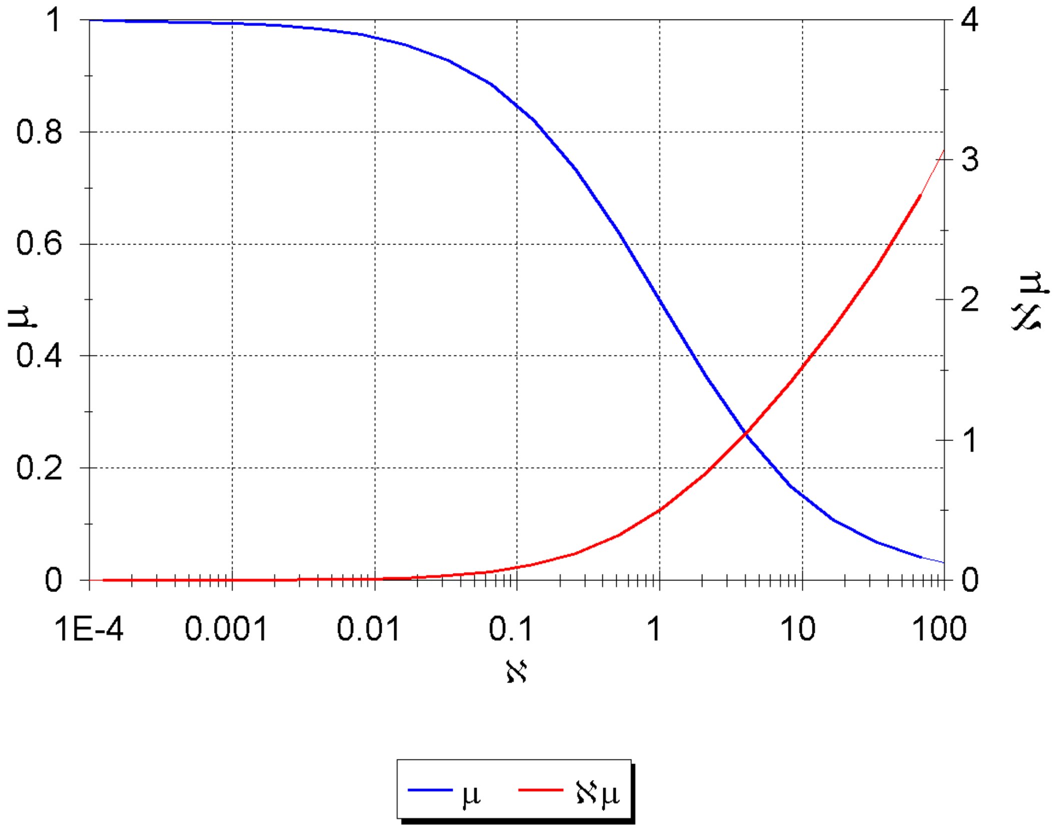
$$\mu = \frac{G}{G_0}$$

$$\aleph = \frac{\gamma}{\gamma_0}$$



$$\tau = \frac{\gamma}{\frac{1}{G_0} + \frac{\gamma}{\tau_0}}$$

Quantifying the Dimensionless Parameters

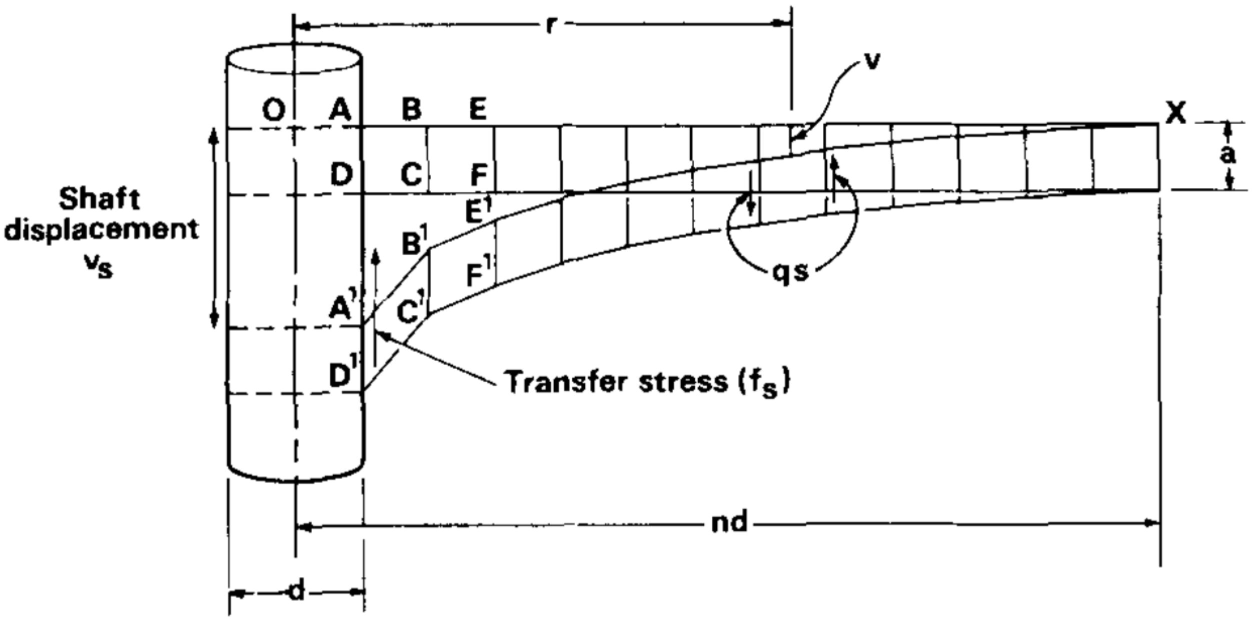


$$\frac{\tau}{\tau_0} = \mu\aleph$$

$$\mu = \frac{\beta}{\beta + \aleph^{\alpha}}$$

Chart assumes β=1, α=0.75

Deflection of Soil Around Deep Foundation



$$w_s = \tau_1 r_1 \int_{r_1}^{r_m} \frac{dr}{G(r)\, r}$$

From Cooke (1974)
"It should be noted that a constant value of shear modulus G in horizontal planes is not essential to this general concept of load development in terms of displacement."

Theory based on static equilibrium, outer radius must be limited to prevent unbounded condition ("magical radius")

Discrete Solution for Hyperbolically Varying Stress-Strain Conditions

$$w_s = \tau_1 r_1 \sum_{n=1}^{k} \frac{\ln(r_{n+1}) - \ln(r_n)}{\mu_{n\rightarrow n+1} G_0}$$

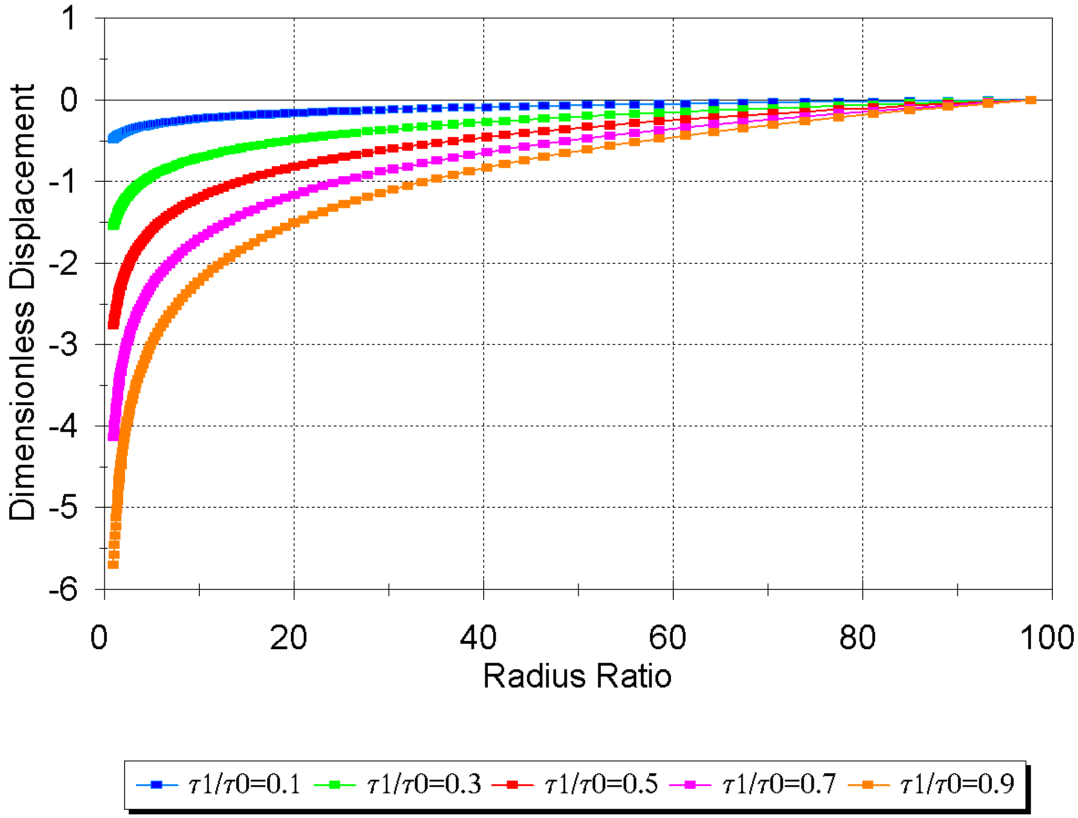
$$\hat{w}_s = \aleph_1 \mu_1 \frac{\ln(\hat{r}_{n+1}) - \ln(\hat{r}_n)}{\mu_n} = \frac{G_0 w_s}{\tau_0 r_1}$$

$$w_s = \frac{\tau_0 r_1}{G_0} \hat{w}_s = \gamma_0 r_1 \hat{w}_s$$

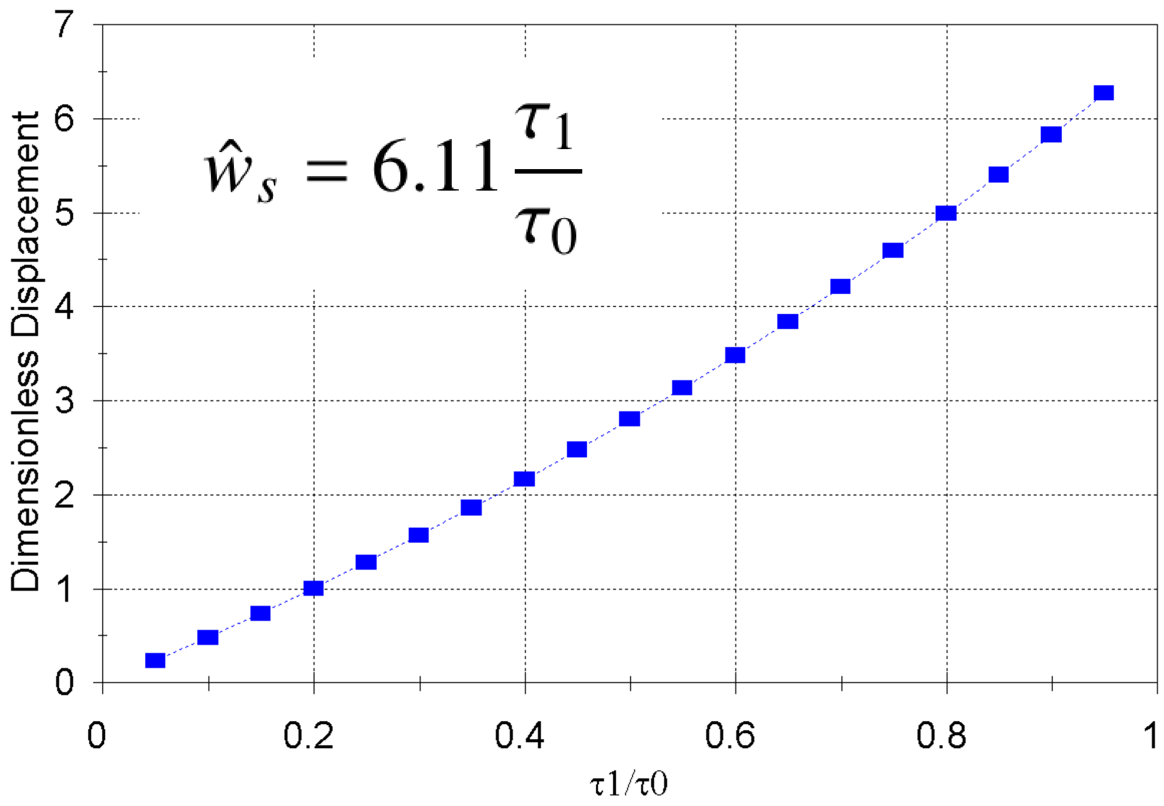
Outline of the Solution

- Establish a range of values of shear strains to analyze
- Assume values of α and β for hyperbolic strain equation
- Assume value of “magical radius” (in our case 100)
- Divide region in to small increments
- Determine stress ratio of each increment (the first point is the pile-soil interface)
- Using Newton’s Method, determine the values of μ and $\aleph$ for each point
- Estimate the average dimensionless deflection for each increment
- Sum up for a total dimensionless deflection for the pile surface relative to the soil

Soil Deflections as a Function of Radius Ratio



Dimensionless Displacement of Pile vs. Shear Stress Ratio



Notes on the Solution

- There are many “loose ends” in this solution. They do not necessarily relate well to existing soil models (Mohr-Coulomb) and many things are assumed to obtain a result
- Although it is strictly speaking only valid for rigid piles, the pile can be divided up and the soil layered to apply it to a multi-layered solution
- The results indicate that the use of elastic-plastic soil models (such as are common in t-z axial load-deflection analyses) may be valid even with the inclusion of hyperbolic soil stress-strain effects before failure of the pile-soil interface
- Dynamic loading will force the inclusion of other factors, including the response of the soil itself

Conclusion

A simple method was developed to include the effects of strain-softening around driven piles due to shear stresses along the shaft. The method focuses on the reduction of the shear modulus to the exclusion of other parameters. For a more practical implementation the parameters necessary to transform the dimensionless results to actual physical ones need proper quantification.

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