VOLTAGE COLLAPSE MODES IN POWER NETWORKS

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ABSTRACT

Voltage Collapse is instability of a heavily loaded electric power system, which leads to declining voltages and blackout, and it is typically associated with reactive power limitations of the power system. Therefore, knowledge of the critical loading point is vital to ensure a secure mode of operation for the power system. Over the years, different methods were proposed in the literature to identify proximity to voltage collapse. However, these techniques were derived utilizing only the voltage limitation of the load buses.

Voltage collapse is often interlinked with static transfer stability limit. In this work, different voltage collapse modes are addressed and a new algorithm that takes into account both load bus voltage and generator bus angle behavior is proposed to estimate the collapse point. The proposed method was examined on several IEEE test systems. Also, superiority of the proposed method in comparison with the other techniques was proven.
TABLE OF CONTENTS

ABSTRACT...................................................................................................................... iv

TABLE OF CONTENTS.................................................................................................... v

LIST OF TABLES............................................................................................................... vii

LIST OF FIGURES............................................................................................................ viii

CHAPTER

1. INTRODUCTION.......................................................................................................... 1

   1.1 Overview.............................................................................................................. 1
   1.2 Problem Statement.............................................................................................. 1
   1.3 Objective.............................................................................................................. 2
   1.4 Thesis Layout...................................................................................................... 2

2. LITERATURE REVIEW.............................................................................................. 3

   2.1 Introduction......................................................................................................... 3
   2.2 Classification of Stability................................................................................... 5
      2.2.1 Voltage Instability...................................................................................... 6
      2.2.2 Angle Instability....................................................................................... 8
   2.3 P-V and Q-V Characteristics............................................................................. 9
   2.4 Modal Analysis.................................................................................................. 12
   2.5 Voltage Collapse Prediction............................................................................. 14
      2.5.1 Line Stability index.................................................................................... 15
      2.5.2 Coupled Single-Port method................................................................. 17
      2.5.3 The L index.............................................................................................. 20
      2.5.4 The P index.............................................................................................. 21

3. METHODOLOGY....................................................................................................... 25

   3.1 Load Bus Voltage Limitation Mode................................................................. 25
      3.1.1 Two Bus System....................................................................................... 25
      3.1.2 General N-Bus System............................................................................. 29
      3.1.3 Determination of Weakest Bus............................................................... 31
      3.1.4 Application of the Voltage Instability Method to Test System............. 32
3.1.5 Modification of Load-Flow for Voltage Mode Method .................. 33
3.2 Static Transfer Stability Limit Mode .................................. 36
  3.2.1 Kron Reduction .................................................. 36
  3.2.2 Necessary Condition for Collapse .............................. 37
  3.2.3 Application of the STSL Method to Test System ............... 38
  3.2.4 Modification of Load-Flow for STSL Mode Method .......... 39

4. RESULTS AND DISCUSSION ................................................. 41
  4.1 Introduction .............................................................. 41
  4.2 Testing the Methods on the IEEE 14-Bus System .................. 41
  4.3 Testing the methods on the IEEE 39-bus system .................. 43
  4.4 Testing the methods on the IEEE 57-bus system ................. 45
  4.5 Testing the methods on the IEEE 118-bus system ............... 46
  4.6 Testing the methods on the IEEE 300-bus system ............... 48

5. CONCLUSION AND FUTURE WORK ....................................... 50
  5.1 Conclusion ............................................................. 50
  5.2 Future Work ............................................................ 51

REFERENCES ................................................................. 52

VITA ........................................................................ 54
LIST OF TABLES

4.1 Comparison between the Voltage Mode and STSL Mode Methods for the IEEE 14-bus System ................................................................. 43

4.2 Comparison between the Voltage Mode and STSL Mode Methods for the IEEE 39-bus System ........................................................................ 44

4.3 Comparison between the Voltage Mode and STSL Mode Methods for the IEEE 57-bus System ...................................................................... 46

4.4 Comparison between the Voltage Mode and STSL Mode Methods for the IEEE 118-bus System ..................................................................... 47

4.5 Comparison between the Voltage Mode and STSL Mode Methods for the IEEE 300-bus System ................................................................. 49
LIST OF FIGURES

2.1 Classification of Power System Stability ......................................................... 6
2.2 Basic Two-Bus Power System ........................................................................... 7
2.3 Receiving End Power, Voltage and Current as Function of Load Demand for the System ... 8
2.4 Typical Power-Angle Curve ............................................................................. 9
2.5 Active Power Loading and Bus Voltage Relationship Curve (P-V) ................. 10
2.6 Continuation Power Flow Predictor-Corrector Scheme ................................. 11
2.7 Reactive Power Injection and Bus Voltage Relationship Curve (Q-V) .............. 12
2.8 Typical One-Line Diagram of Transmission Line ............................................. 16
2.9 Multi-Port Network System Model .................................................................. 17
2.10 Graphical Representation of Multi-Port Thévenin Equivalent ......................... 19
2.11 Two Bus System ............................................................................................ 22
3.1 Two Bus Network ........................................................................................... 26
3.2 Bus-2 Voltage2 and V-axis ............................................................................. 28
3.3 Power System Aggregation Based on Power Importing Buses .......................... 31
3.4 Bus-5 Voltage 2 and Axis Voltage ................................................................. 32
3.5 Bus-5 Voltage 2 and Axis Voltage for Case 2-3 Outage ................................. 33
3.6 Branches Static Transfer Stability Limit for Line 2-3 Outage ......................... 38
3.7 Branches Static Transfer Stability Limit for Line 13-14 Outage ....................... 39
4.1 Single Line Diagram of the IEEE 14-Bus System ........................................... 42
4.2 Single Line Diagram of the IEEE 39-Bus System................................................................. 44
4.3 Single Line Diagram of the IEEE 57-Bus System........................................................................ 45
4.4 Single Line Diagram of the IEEE 118-Bus System..................................................................... 47
4.5 Single Line Diagram of the IEEE 300-Bus System..................................................................... 48
CHAPTER 1
INTRODUCTION

1.1 Overview

Voltage collapse has become one of the important issues in today’s highly developed power systems as several major blackouts in recent years could be traced to voltage collapse. Past incidents of voltage collapse have caused millions of dollars of equipment damage and have produced service interruptions to thousands of customers at a time. One method to prevent voltage collapse from occurring requires online voltage monitoring tools to predict the point of collapse and make corrective actions before the system enters critical condition. However, accurate estimation of the voltage collapse point proves to be a challenge.

1.2 Problem Statement

Several methods that determine the maximum loading point of a power system have been proposed in the literature since it serves as a principal voltage stability measurement [1]. However, the existing solution methods and indices show inaccuracies and they only solve the voltage collapse problem as a load-bus voltage stability driven problem. It is quite evident that there is still a need of an improved approach, which also looks at the angles of the generator buses and static transfer stability limit of network branches for potential links to voltage collapse assessment.
1.3 Objective

The first objective of this work is to develop a new fast and simple method to calculate the maximum loading point based on the voltage instability of the load buses using basic power flow equations. The second objective is to alternatively obtain the maximum loading point utilizing the angle behavior of the generator buses and the static transfer stability limit of individual branches.

1.4 Thesis Layout

The remainder of this thesis is organized as follows:

- Chapter II: this chapter provides an overview of the literature on different voltage stability analysis tools.
- Chapter III: this chapter introduces the concepts behind the proposed method along with the derivation of their formulas.
- Chapter IV: this chapter presents simulation results when applying the proposed methods on different test systems. Moreover, a discussion on the performance of the proposed method is presented.
- Chapter V: this chapter concludes the contributions and findings of this work. Furthermore, it provides suggestions and recommendations for future research work.
CHAPTER 2
LITERATURE REVIEW

2.1 Introduction

The definition of voltage stability as proposed by the IEEE/CIGRE Task Force is as follows: “Voltage stability is the ability of a power system to maintain steady voltages at all buses in the system after being subjected to a disturbance from a given initial operating condition” [2]. Voltage stability events span a time ranging from a few cycles to minutes. Based on different time frames, voltage stability can be classified into transient voltage stability and long-term voltage stability. The time frame of transient voltage stability is zero to ten seconds, while the time frame of long-term voltage stability is often several minutes [3].

A power system may be subject to voltage instability when a disturbance, an increase in load demand or alteration in system state causes a progressive and uncontrollable drop in system voltage [3]. It is highly influenced by transmission system characteristics, generator characteristics and load dynamics:

- Transmission system characteristics: Transfer of active and reactive power is provided by transmission lines. Since transmission lines are generally long, transfer of reactive power over these lines is very difficult due to significant amount of reactive power requirement and limited when the load on transmission lines is too high and/or the voltage sources are too far from the load centers.
Generator characteristics: Under normal conditions the terminal voltages of generators are maintained constant. During conditions of low-system voltages, the reactive power demand on generator may exceed their field current and/or armature current limits. When the reactive power output is limited, the terminal voltage is no longer maintained constant.

Load dynamics: Stable operation of power system depends on the ability to continuously match the electrical output of generating units to the electrical load on the system. The problem of maintaining voltages within the required limits is complicated by the fact that the power system supplies power to a large number of loads and is fed from many generating units. As loads vary, the reactive power requirements of the transmission system vary. When the voltage starts to drop after a disturbance, constant power loads such as industrial motor loads, air conditioner, etc. tend to maintain their active power consumption through the action of motor slip adjustment, distribution voltage regulators, thermostats, etc. This would result in increasing the reactive power consumption which would cause the voltage to drop much further.

The term voltage collapse is also often used for voltage instability conditions. According to IEEE/CIGRE Joint Task Force, voltage collapse is “the process by which the sequence of events accompanying voltage instability leads to a blackout or abnormally low voltages in a significant part of the power system” [2]. Hill et al, in their set of stability definitions [4] define voltage collapse as “a power system at a given operating state and subject to a given large disturbance undergoes voltage collapse if it is voltage unstable or the post-disturbance equilibrium values are nonviable”. Investigation of recent blackout events indicates that the root
cause of several major network collapses is voltage instability. The following are some examples [3], [5]:

- Florida system disturbance of December 28, 1982.
- Northern Belgium system disturbance of August 4, 1982.

2.2 Classification of Stability

The classification of power system stability proposed here is based on the following considerations [3]:

- The physical nature of the resulting mode of instability.
- The size of the disturbance considered which influences the method of calculation and prediction of stability.
- The devices, processes, and time span that must be taken into consideration in order to assess stability.

Figure 2.1 gives the overall picture of the power system stability problem, identifying its categories and subcategories and the following section describes the corresponding forms of stability phenomena.
2.2.1 Voltage Instability

A simple two-bus power system consisting of a voltage source ($E_s$), a load ($Z_D$), and a purely reactive transmission line ($Z_L$) as shown in figure 2.2 can be used to illustrate the voltage instability problem. The magnitude of the current flowing through the circuit is governed by the equation shown below.

$$I = \frac{E_s}{\sqrt{(Z_L \cos \theta + Z_D \cos \phi)^2 + (Z_L \sin \theta + Z_D \sin \phi)^2}}$$

The receiving end voltage can be expresses as:
\[ V_R = Z_D I \]  \hspace{1cm} (2.2)

While the active power drawn by the load is given by:

\[ P_R = V_R I \cos \phi \]  \hspace{1cm} (2.3)

The real and reactive power of the load can be increased by decreasing the load impedance \( Z_D \) while maintaining a constant power factor. The current will increase due to decrease in the load impedance, even though the receiving end voltage magnitude will drop hence the real power consumed by the load will increase. Once operating point reaches the maximum power point or knee point of the curve any further decrease in load impedance, to increase the load, will result in further drop in the receiving end voltage and simultaneously the real power consumed by the load will also decrease. Controlling the load power past the maximum point is unstable since a decrease in load impedance would result in reducing active power. Thus, the knee point is where the system reaches a maximum tolerable voltage difference between the load and source. Figure 2.3 demonstrate the normalized values of \( I, V_R \) and \( P_R \) versus \( Z_D/Z_L \)and it is evident that the receiving end power increases as the load
decreases until it reaches the maximum; after that the power changes its pattern and starts to decrease.

![Graph of Z_L/Z_D vs P_R/P_RMAX, V_R/E_s, I/I_SC]

Figure 2.3
Receiving End Power, Voltage and Current as Function of Load Demand for the System

2.2.2 Angle Instability

Assuming the voltage magnitude of the source and load are held constant, and the load voltage angle is fixed at 0 for a lossless line of reactance \( X \), the power delivered to the load is governed by the following equation

\[
P_L = \frac{E_s V}{X} \sin \delta
\]  

As shown in figure 2.4 the active power varies as a sine of the angle; the source angle increase as the active power transferred to the load increase, but the power transfer reaches a
maximum after a certain angle, nominally 90°. The idea of angle instability can be further illustrated, by understanding the operation of synchronous machine of rotor angle $\delta$. If the load increases above the maximum shown on the curve, the rotor angle would advance as the machine attempts to serve the additional load. However, any angle displacement above certain angle causes a decrease in the machine output instead of an increase, and the angle advance would be a fruitless effort. Thus, angle instability is similar to voltage instability in that it imposes a limit on the power that can be transmitted to the load, but the constraint is angle-oriented instead of voltage-oriented.

![Typical Power-Angle Curve](image)

**Figure 2.4**

Typical Power-Angle Curve

### 2.3 P-V and Q-V Characteristics

P-V and Q-V curves are amongst the most fundamental power flow-based static analysis tools. P-V curve analysis is used to determine voltage stability of a radial system and also a large meshed network; it shows the relationship between the power injection and the corresponding change in voltage at a particular bus. For this analysis power (P) at a particular area is increased in steps and voltage (V) is observed at some critical load buses and then the $V - P$ curves for
those particular buses will be plotted to determine the voltage stability of a system by static
analysis approach. These curves are known as nose curves. When approaching the voltage
collapse point, voltage decreases drastically with the slightest increase in load and the Jacobian
matrix of power flow equations becomes singular and the regular power flow solution does not
converge. For this reason, the continuation power flow was developed to overcome this problem
by reformulating the power flow equations so that they remain well-conditioned at all possible
loading conditions [6]. This allows the solution of power-flow problem for stable as well as
unstable equilibrium point [3].

The continuation load flow method uses two basic stages including predictor stage and
corrector stage. The predictor stage finds an approximation for the next solution. The exact
solution is then obtained by performing a conventional power flow in a corrector step. After that,
a new prediction is made for another increase in load based on the new tangent vector. Then
corrector step is applied. The predictor-corrector process is performed until the critical point is
reached. At the critical point the tangent vector is zero [7]. Figures 2.5 and 2.6 illustrate the P-V
curve and the Predictor-Corrector scheme respectively.

![Figure 2.5](image)

**Figure 2.5**
Active Power Loading and Bus Voltage Relationship Curve (P-V)
The second tool for analyzing voltage stability is the Q-V curves. These curves show the sensitivity and variation of bus voltages with respect to reactive power injections. The collapse point is where the rate of change in reactive power with respect to voltage is equal to zero and represents the voltage stability limit. The right-hand side of the curve is stable since an increase in Q is accompanied by an increase in V. The other side is unstable where an increase in Q represents a decrease in V as shown in figure 2.7.

P-V and Q-V curves are useful in estimating the distance to voltage collapse. However, the main disadvantage of these curves is the fact that for many different operating points and contingencies, a large number of such curves would be required to obtain complete information on the voltage stability of the whole system. Each one of those curves has to undergo a large number of power flow solutions. This makes them very time-consuming and hence not practical for on-line voltage stability monitoring of large power systems.
2.4 Modal Analysis

B. Gao and P. Kundur proposed a modal analysis approach to evaluate voltage stability for large power systems in 1992 [8]. Based on a linear approximation of the system model, modal analysis calculates the eigenvalue and eigenvector of the reduced Jacobian matrix. Each eigenvalue represents a mode of $V$-$Q$ variation. The magnitude of the eigenvalue can be considered as a quantitative measurement of the static voltage stability margin. The eigenvectors are used to calculate the bus participation factors which indicate the weak areas of the system.

The linearized steady state power systems equations can be expressed as:

$$
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} = \begin{bmatrix}
J_{P\delta} & J_{PV} \\
J_{Q\delta} & J_{QV}
\end{bmatrix} \begin{bmatrix}
\Delta \delta \\
\Delta V
\end{bmatrix}
$$

System voltage stability is affected by both $P$ and $Q$. However, $Q$-$V$ sensitivity analysis is done by keeping real power $P$ constant at each operating point and evaluating voltage stability by considering the incremental relationship between $Q$ and $V$.

Let $\Delta P = 0$, then:
\[ \Delta P = 0 = J_{P\delta} \Delta \delta + J_{PV} \Delta V \quad 2.6 \]
\[ \Delta Q = J_{Q\delta} \Delta \delta + J_{QV} \Delta V \quad 2.7 \]

Substituting \( \Delta \delta \) in equation 2.7:

\[ \Delta Q = (J_{QV} - J_{Q\delta} \cdot J_{P\delta}^{-1} \cdot J_{PV}) \Delta V \quad 2.8 \]

The reduced Jacobian matrix \( J_R \) can be defined as:

\[ J_R = [J_{QV} - J_{Q\delta} \cdot J_{P\delta}^{-1} \cdot J_{PV}] \quad 2.9 \]

Equation 2.8 becomes:

\[ \Delta Q = J_R \Delta V \quad 2.10 \]

In the Q-V analysis, equation 2.10 can be written as:

\[ \Delta V = J_R^{-1} \Delta Q \quad 2.11 \]

The decomposition of \( J_R \) and \( J_R^{-1} \) are:

\[ J_R = \xi \Lambda \eta \quad 2.12 \]
\[ J_R^{-1} = \xi \Lambda^{-1} \eta \quad 2.13 \]

Where:

\( \xi = \) right eigenvector matrix of the reduced jacobian matrix
\( \eta = \) left eigenvector matrix of the reduced jacobian matrix
\( \Lambda = \) diagonal eigenvalue matrix of the reduced jacobian matrix

Equation 2.11 can be written as:

\[ \Delta V = \xi \Lambda^{-1} \eta \Delta Q \quad 2.14 \]

Or
\[ \Delta V = \sum_i \frac{\xi_i \eta_i}{\lambda_i} \Delta Q \quad 2.15 \]

The \(i^{th}\) mode of Q-V response is defined by the eigenvalue \(\lambda_i\) and the corresponding right and left eigenvectors \(\xi_i\) and \(\eta_i\).

Equation 2.15 describes the Q-V response of each mode. The sign and magnitude of \(\lambda_i\) provide a qualitative measure of system stability. A positive \(\lambda_i\) indicates that the incremental change in voltage magnitude of bus \(i\) is along the direction of the incremental change in reactive power injection at bus \(i\). Hence, the system is at a stable operating condition if \(\lambda_i\) is positive. A negative \(\lambda_i\) indicates that the incremental change in voltage magnitude of bus \(i\) is along the opposite direction of the incremental change in reactive power injection at bus \(i\). Hence, the system is at an unstable operating condition if \(\lambda_i\) is negative. A value of \(\lambda_i = 0\) indicates a voltage collapse since any variation in reactive power injection gives infinite change in voltage magnitude.

### 2.5 Voltage Collapse Prediction

In literature, different types of voltage stability indices have been introduced in order to evaluate the stability limit. Voltage stability indices are invaluable tools for gauging the proximity of a given operating point to voltage collapse. The objective of the voltage stability indices is to quantify how close a particular point is to the steady state voltage stability margin. These indices can be used on-line or offline to help operators in real time operation of power system or in designing and planning operations. According to [9],[10] it is mentioned that voltage stability indices particularly can be classified into Jacobian matrix based voltage stability indices and system variables based voltage stability indices. Jacobian matrix based voltage stability indices are able to calculate the voltage collapse point of the system and discover the
voltage stability margin. However, some of these indices – particularly those that use modal analysis - require high computational time and for this particular reason, these Jacobian matrix based voltage stability indices may not be appropriate for online assessment. Meanwhile, system variables based voltage stability indices required less computational time since they typically only use the elements of the admittance matrix and some system variables such as bus voltages or power flow through the lines. With the benefit of less computational time, system variables based voltage stability indices are suitable to be implemented for online assessment and monitoring purposes. However, many of these system variables based voltage stability indices are not very accurate and fail to capture the non-linear behavior involved in voltage stability. Some of them capture local singularities related to a line or a group of lines, which is not aligned with system-wide collapse.

A brief overview of different indices will be presented in this section with the general aim of estimating how close to voltage collapse a system can be operated to avoid a blackout in large parts of the interconnection.

2.5.1 Line Stability index

Based on a power transmission concept in a single line, Moghavvemi derived a voltage stability index [11]. Consider a single line of interconnected network and the line is represented with the following parameters shown in figure 2.8:
Figure 2.8

Typical One-Line Diagram of Transmission Line

Utilizing the concept of power flow in the line and analyzing with pi-model representation, the power flow at the sending and receiving end can be expressed as:

\[ S_r = \frac{|V_s||V_r|}{Z} \angle (\theta - \delta_1 + \delta_2) - \frac{|V_r|^2}{Z} \angle \theta \]  \hspace{1cm} 2.16

\[ S_s = \frac{|V_s|^2}{Z} \angle \theta - \frac{|V_s||V_r|}{Z} \angle (\theta + \delta_1 - \delta_2) \]  \hspace{1cm} 2.17

From the above equations, active and reactive power can be separated as follows:

\[ P_r = \frac{|V_s||V_r|}{Z} \cos(\theta - \delta_1 + \delta_2) - \frac{|V_r|^2}{Z} \cos \theta \]  \hspace{1cm} 2.18

\[ Q_r = \frac{|V_s||V_r|}{Z} \sin(\theta - \delta_1 + \delta_2) - \frac{|V_r|^2}{Z} \sin \theta \]  \hspace{1cm} 2.19

Putting \( \delta_1 + \delta_2 = \delta \) into equation 2.19 and solving for \( V_r \):

\[ V_r = \frac{V_s \sin(\theta - \delta) \pm \sqrt{[V_s \sin(\theta - \delta)]^2 - 4Z Q_r \sin \theta}}{2 \sin \theta} \]  \hspace{1cm} 2.20

For \( Z \sin \theta = x \), we have

\[ V_r = \frac{V_s \sin(\theta - \delta) \pm \sqrt{[V_s \sin(\theta - \delta)]^2 - 4x Q_r}}{2 \sin \theta} \]  \hspace{1cm} 2.21
To obtain real values for $V_r$, the discriminator of equation 2.21 must be greater than or equal to zero. Thus the following conditions, which can be used as a stability criterion, need to be satisfied:

$$\{[V_s \sin(\theta - \delta)]^2 - 4xQ_r \} \geq 0 \quad 2.22$$

Or:

$$\frac{4xQ_r}{[V_s \sin(\theta - \delta)]^2} = L_{mn} \leq 1 \quad 2.23$$

$L_{mn}$ is termed as the line stability index and any line in the system that exhibits line stability index closed to one indicates that the line is approaching its stability limit. Lines that present values of $L_{mn}$ less than 1, indicates that the system is stable and when $L_{mn}$ is greater than 1, the system loses its stability and voltage collapse occurs.

### 2.5.2 Coupled Single-Port method

Voltage collapse detection based on the concept of coupled single-port circuit considers all loads in the system as constant-power type loads. One interesting approach based on this concept is proposed in [12]. In this approach, when monitoring a particular load bus, all other loads are represented into system equivalence impedance. This equivalent impedance $Z_L$, shown in figure 2.9, leads to the concept of multi-port network equivalent.

![Multi-Port Network System Model](image)

Figure 2.9

Multi-Port Network System Model
In matrix form, system in figure 2.9 can be described as follows:

\[
\begin{bmatrix}
-I_L \\
0 \\
I_G
\end{bmatrix}
= [Y]
\begin{bmatrix}
V_L \\
V_T \\
V_G
\end{bmatrix}
= \begin{bmatrix}
Y_{LL} & Y_{LT} & Y_{LG} \\
Y_{TL} & Y_{TT} & Y_{TG} \\
Y_{GL} & Y_{GT} & Y_{GG}
\end{bmatrix}
\begin{bmatrix}
V_L \\
V_T \\
V_G
\end{bmatrix}
\]

2.24

Where the Y matrix is known as the system admittance matrix, V and I stand for the voltage and current vectors, and the subscript L, T and G represent load bus, tie bus, and generator bus, respectively.

From the matrix above, the expression of \( V_L \) can be obtained as follows:

\[ Y_{LL}V_L = -Y_{LT}V_T - Y_{LG}V_G - I_L \]

2.25

In addition, \( V_T \) can be expressed, as follows:

\[ V_T = Y_{TT}^{-1}(-Y_{TL}V_L - Y_{TG}V_G) \]

2.26

By integrating equation 2.25 and equation 2.26, the following expression is obtained:

\[ (Y_{LL} - Y_{LT}Y_{TT}^{-1}Y_{TL})V_L = (Y_{LT}Y_{TT}^{-1}Y_{TG} - Y_{LG})V_G - I_L \]

2.27

By calling \( Z_L = (Y_{LL} - Y_{LT}Y_{TT}^{-1}Y_{TL})^{-1} \), equation 2.27 can be written as:

\[ V_L = Z_L(Y_{LT}Y_{TT}^{-1}Y_{TG} - Y_{LG})V_G - Z_LI_L \]

2.28

Finally, if we define \( K = Z_L(Y_{LT}Y_{TT}^{-1}Y_{TG} - Y_{LG}) \), \( V_L \) in the above equation can be written as:

\[ V_L = KV_G - Z_LI_L \]

2.29

From the above equations, for load bus \( j \), \( V_{Lj} \) can be obtained as follows:

\[ V_{Lj} = E_{eqj} - Z_{eqj}I_{Lj} - E_{coupledj} \]

2.30

Where:
\[ E_{eqj} = \sum_{i=1}^{n_G} K_{ji} V_{Gi} \]  

2.31

\[ Z_{eqj} = Z_{Ljj} \]  

2.32

\[ E_{coupledj} = \sum_{i \neq j; i=1}^{n_L} Z_{Lji} I_{Li} \]  

2.33

Where \( n_G \) and \( n_L \) represent the number of generators and loads respectively. \( Z_{eqj} \) is the Thévenin impedance of the network at bus \( j \), it is the diagonal element of the impedance matrix \( Z_L \), and is essentially equal to the short-circuit impedance seen at bus. It remains constant as long as the system topology is the same. \( E_{coupledj} \) represents the impact of other loads on bus \( j \), called the coupling effect.

Using the model in figure 2.10, a power system can be represented by a set of single-port circuits that have the impact of other loads included explicitly. This new equivalent circuit is called “coupled single-port circuit”.

![Figure 2.10](image)

Graphical Representation of Multi-Port Thévenin Equivalent
Now, the issue is how to model the coupling term while maintaining the behavior of the single-port structure. Therefore, modeling \( E_{coup} \) as extra impedance has been proposed by Wang et al. (2011), the maximum power to the load is given by the following expression:

\[
S_{\text{max}} = \frac{|E_{eq}|^2 \left| \left[ Z_{eq} - (\text{imag}(Z_{eq}) \sin \delta + \text{real}(Z_{eq}) \cos \delta) \right] \right|^2}{2 \left( \text{imag}(Z_{eq}) \cos \delta - \text{real}(Z_{eq}) \sin \delta \right)^2} \tag{2.34}
\]

### 2.5.3 The L index

P. Kessel and H. Glavitsch developed in [13] a voltage stability index based on the solution of the power flow equations. The L index is a quantitative measure for the estimation of the distance of the actual state of the system to the stability limit and well suited for online applications.

The transmission system itself is linear and allows a representation in terms of a hybrid (H) matrix

\[
\begin{bmatrix} V_L \\ I_G \end{bmatrix} = [H] \begin{bmatrix} I_L \\ V_G \end{bmatrix} = \begin{bmatrix} Z_{LL} & F_{LG} \\ K_{GL} & Y_{GG} \end{bmatrix} \begin{bmatrix} I_L \\ V_G \end{bmatrix} \tag{2.35}
\]

Where:

- \( V_L, I_L \) = Vector of voltages and currents of the load buses.
- \( V_G, I_G \) = Vectors of voltages and currents of the generator nodes.
- \( Z_{LL}, F_{LG}, K_{GL}, Y_{GG} \) = submatrices of the H-matrix.
- \( F_{LG} \) is the matrix of interest for calculating the L-index and it can be found from the system Y-matrix as follows

\[
F_{LG} = -[Y_{LL}]^{-1}Y_{LG} \tag{2.36}
\]

For any load bus \( j \in L \), the voltage of the bus is known as:
\[ V_j = \sum_{i \in L} Z_{ji} \cdot I_j + \sum_{i \in G} E_{ji} \cdot V_i \]

And:

\[ V_{0j} = -\sum_{i \in G} F_{ji} \cdot V_i \]

Using the above representation, the L index is defined for any load node j will be easily obtained as follows:

\[ L_j = \left| 1 + \frac{V_{0j}}{V_j} \right| = \left| \frac{S_j^*}{Y_{jj} \cdot V_j^2} \right| \]

The above description tells us that L index will get close to 1.0 when a load bus approaches the point of collapse. The concept is extended to a general n bus system through an analogy with the two-bus system, making it non-rigorous.

### 2.5.4 The P index

A new voltage stability indicator called P index was developed to quantify proximity to voltage collapse for on-line assessment of the system voltage stability [14]. The P index is also used to identify the weak bus or buses in the system. Its value varies from 0 at no load to 1.0 at the system point of collapse.

The starting point for the subsequent analysis is a simple radial system. It is given by figure 2.11 where the load at bus 2 is \( P_L + jQ_L \) and the voltage magnitude is \( V \). The equivalent load admittance is \( G_L - jB_L \), where:

\[ G_L = \frac{P_L}{V^2}, \quad B_L = \frac{Q_L}{V^2} \]
If the network load increased to $P_L + \Delta P_L$ and $Q_L + \Delta Q_L$ while keeping its power factor constant, the corresponding load admittance changes to $G_L + \Delta G_L$ and $B_L + \Delta B_L$. The increase in loading will cause the voltage to drop by $\Delta V$. The change in active power can be expressed as:

$$\Delta P_L = (V + \Delta V)^2 (G_L + \Delta G_L) - V^2 G_L$$

$$= (V + \Delta V)^2 \Delta G_L + (2V + \Delta V) G_L \Delta V$$ \hspace{1cm} 2.41$$

The net power is a combination of two opposing terms. The positive term $(V + \Delta V)^2 \Delta G_L$ in equation 2.41 represents the power gained due to the increase of the load $\Delta G_L$, while the negative term $(2V + \Delta V) G_L \Delta V$ is the power lost on the original load $G_L$ due to the drop in voltage $\Delta V$. At the maximum net power delivered to load bus, the two opposing terms cancel each other.

The P index is based on the fact that the two terms in equation 2.41 tends to be close to each other at the maximum power point; it is based on the ratio of the two terms. The negative sign is used to make the index positive when there is a negative voltage drop for positive $\Delta G_L$:

$$P_{index} = -\frac{(2V + \Delta V) G_L \cdot \Delta V}{(V + \Delta V)^2 \cdot \Delta G_L}$$ \hspace{1cm} 2.42$$

In the limiting case as $\Delta G_L, \Delta V \to 0$, the P-index equation (2.45) can be expressed as:
\[ P_{\text{index}} = -\frac{2G_L}{V} \frac{dV}{dG_L} \]  
\[ 2.43 \]

The term \( \frac{dV}{dG_L} \) can be expressed in a more common terms in network as follows:

\[ \frac{dV}{dG_L} = \frac{dV}{dP_L} \frac{dP_L}{dG_L} \]  
\[ 2.44 \]

From 2.40 it can be stated that:

\[ dP_L = V^2 dG_L + 2V G_L dV \]  
\[ 2.45 \]

Or:

\[ \frac{dP_L}{dG_L} = V^2 + 2V G_L \frac{dV}{dG_L} \]  
\[ 2.46 \]

Substituting in 2.44:

\[ \frac{dV}{dG_L} = \frac{dV}{dP_L} \left( V^2 + 2V G_L \frac{dV}{dG_L} \right) \]  
\[ 2.47 \]

Equation 2.47 can be expressed in a different format:

\[ \frac{dV}{dG_L} = \frac{V^2} {1 - 2VG_L \frac{dV}{dP_L}} \]  
\[ 2.48 \]

Substituting 2.48 in the P-index formula:

\[ P_{\text{index}} = \frac{-2VG_L \frac{dV}{dP_L}} {1 - 2VG_L \frac{dV}{dP_L}} \]  
\[ 2.49 \]

Using 2.43, P-index formula can be expressed in terms of active power as follows:

\[ P_{\text{index}} = \frac{-2P_L \frac{dV}{dP_L}} {1 - 2P_L \frac{dV}{dP_L}} \]  
\[ 2.50 \]
As shown above, this index is based on normalized voltage and power sensitivities and therefore, it conveys a better estimate of absolute stability in comparison to other indices. Furthermore, the proposed index is intuitive and its value ranges between 0 for no load conditions and 1 at the point of collapse.
CHAPTER 3

METHODOLOGY

The method proposed in this work uses two different approaches to estimate the voltage collapse point. The first approach is based on the voltage instability of the load buses. The other one is inspired by angle limitation of the generator buses and static transfer stability limit of branches. Both approaches are tested for each system to find the maximum net power delivered and two solutions are obtained. Since both methods provide solution of the maximum loading point not prediction; maximum load of the system is the maximum of the two solutions. It must be mentioned that the network load and generation be incrementally increased in the same proportion until the maximum load-multiplier point $\lambda_{\text{max}}$.

3.1 Load Bus Voltage Limitation Mode

This method is proposed to assess the voltage collapse in a system through examining the voltage behavior of the load buses and it is based on the basic load flow equations at bus $i$ for a given loading condition and it is formulated by using the elements of the bus admittance matrix.

3.1.1 Two Bus System

First let us consider a two bus power system as shown in figure 3.1 where node $i$ is assumed to supply the load whereas node $j$ is the generator node.
The active and reactive power at a bus $i$ in a power system network can be presented as

$$P_i = P_{gi} - P_{di} = \sum_{j=1}^{n} V_i V_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij})$$  \hspace{1cm} 3.1$$

$$Q_i = Q_{gi} - Q_{di} = \sum_{j=1}^{n} V_i V_j Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij})$$  \hspace{1cm} 3.2$$

The active power equations at the load bus $i$ for the two bus network as shown in figure 3.1 can be brought into the form:

$$P_i = V_i^2 G_{ii} + V_i V_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij})$$  \hspace{1cm} 3.3$$

$$P_i - V_i^2 G_{ii} = V_i V_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij})$$  \hspace{1cm} 3.4$$

Similarly, reactive power equations will be presented as follows:

$$Q_i = -V_i^2 B_{ii} + V_i V_j Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij})$$  \hspace{1cm} 3.5$$

$$Q_i + V_i^2 B_{ii} = V_i V_j Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij})$$  \hspace{1cm} 3.6$$

Squaring equation 3.4 and equation 3.6 gives,
\[ P_i^2 + V_i^4 G_{ii}^2 - 2P_i V_i^2 G_{ii} = (V_i V_j Y_{ij})^2 \cos^2 (\delta_i - \delta_j - \theta_{ij}) \quad 3.7 \]

\[ Q_i^2 + V_i^4 B_{ii}^2 + 2Q_i V_i^2 B_{ii} = (V_i V_j Y_{ij})^2 \sin^2 (\delta_i - \delta_j - \theta_{ij}) \quad 3.8 \]

Adding equation 3.7 and 3.8 will give,

\[
P_i^2 + Q_i^2 + V_i^4 (G_{ii}^2 + B_{ii}^2) - 2(P_i G_{ii} - Q_i B_{ii}) V_i^2
= (V_i V_j Y_{ij})^2 \cos^2 (\delta_i - \delta_j - \theta_{ij}) + (V_i V_j Y_{ij})^2 \sin^2 (\delta_i - \delta_j - \theta_{ij}) \quad 3.9
\]

Making the substitutions \( Y_{ii}^2 = G_{ii}^2 + B_{ii}^2 \) and \( S_i^2 = P_i^2 + Q_i^2 \), equation 3.9 may be rearranged as,

\[
V_i^4 - 2 \left( \frac{(P_i G_{ii} - Q_i B_{ii})}{Y_{ii}^2} + \frac{(V_i Y_{ij})^2}{2 Y_{ii}^2} \right) V_i^2 + \frac{S_i^2}{Y_{ii}^2} = 0 \quad 3.10
\]

Whereby an equivalent voltage \( V_0^2 \) is substituted as,

\[
V_0^2 = \frac{(V_i Y_{ij})^2}{Y_{ii}^2} \quad 3.11
\]

Therefore equation 3.10 may be written as:

\[
V_i^4 - 2 \left( \frac{(P_i G_{ii} - Q_i B_{ii})}{Y_{ii}^2} + V_0^2 \frac{2}{Y_{ii}^2} \right) V_i^2 + \frac{S_i^2}{Y_{ii}^2} = 0 \quad 3.12
\]

The solution of equation 3.12 is clearly defined by,

\[
V_i^2 = \frac{(P_i G_{ii} - Q_i B_{ii})}{Y_{ii}^2} + V_0^2 \frac{2}{Y_{ii}^2} \pm \sqrt{\left( \frac{(P_i G_{ii} - Q_i B_{ii})}{Y_{ii}^2} + \frac{V_0^2}{2} \right)^2 - \frac{S_i^2}{Y_{ii}^2}} \quad 3.13
\]
To obtain a real solution of $V_i^2$, \( \left[ \left( \frac{P_i G_{ii} - Q_i B_{ii}}{Y_{ii}^2} \right) + \frac{V_0^2}{2} \right] - \frac{S_i^2}{Y_{ii}^2} \geq 0 \) should be true; we shall take the positive square as the feasible solution. Maximum net power delivered to load bus $i$ is reached when the discriminator term vanishes, so voltage collapse point is the point when the square of the voltage at bus $i$ equals to the first term in equation 3.13. In this work, the first term \( \left( \frac{P_i G_{ii} - Q_i B_{ii}}{Y_{ii}^2} \right) + \frac{V_0^2}{2} \) is named the ‘axis of $V_i^2$', since it describes an axis of the parabola tracing $V_i^2$.

It is not a difficult matter to obtain the above value of axis voltage; the calculation involves the admittance matrix and the active and reactive power of the load bus $i$. A plot of the square of load bus voltage $V$ versus load multiplier for two bus system is shown in figure with $E=1.0$ p.u, $Z=j0.2$ p.u. On the same plot the corresponding parabola ‘axis’ which represents the axis voltage is drawn and it is clear that the voltage collapse occurs at the point where the bus voltage intersects with the axis voltage for the load bus (bus-2).

Figure 3.2
Bus-2 Voltage$^2$ and V-axis
3.1.2 General N-Bus System

Equation 3.1 and 3.2 may then be used as the basis to obtain the voltage solution for n-bus system, similar to the derivations carried out for two-bus system. If the power injection equations are written as

\[ P_i - V_i^2 G_{ii} = \sum_{j=1, j\neq i}^{n} V_i V_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij}) \]  
\[ Q_i + V_i^2 B_{ii} = \sum_{j=1, j\neq i}^{n} V_i V_j Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij}) \]

By taking the terms involving \( i \) out of the summation, equation 3.14 and 3.15 may be expressed in the following forms:

\[ P_i - V_i^2 G_{ii} = V_i \cos \delta_i \sum_{j=1, j\neq i}^{n} V_j Y_{ij} \cos(\delta_j + \theta_{ij}) + V_i \sin \delta_i \sum_{j=1, j\neq i}^{n} V_j Y_{ij} \sin(\delta_j + \theta_{ij}) \]  
\[ Q_i + V_i^2 B_{ii} = V_i \sin \delta_i \sum_{j=1, j\neq i}^{n} V_j Y_{ij} \cos(\delta_j + \theta_{ij}) - V_i \cos \delta_i \sum_{j=1, j\neq i}^{n} V_j Y_{ij} \sin(\delta_j + \theta_{ij}) \]

Let \( \sigma_{1i} = \sum_{j=1, j\neq i}^{n} V_j Y_{ij} \cos(\delta_j + \theta_{ij}) \) and \( \sigma_{2i} = \sum_{j=1, j\neq i}^{n} V_j Y_{ij} \sin(\delta_j + \theta_{ij}) \),

We may reduce 3.16 and 3.17 to,

\[ P_i - V_i^2 G_{ii} = V_i \cos \delta_i \sigma_{1i} + V_i \sin \delta_i \sigma_{2i} \]  
\[ Q_i + V_i^2 B_{ii} = V_i \sin \delta_i \sigma_{1i} - V_i \cos \delta_i \sigma_{2i} \]

Squaring equation 3.18 and 3.19 and adding them together will result in equation 3.20.

\[ P_i^2 + Q_i^2 - 2(P_i G_{ii} - Q_i B_{ii})V_i^2 + (G_{ii}^2 + B_{ii}^2)V_i^4 = V_i^2(\sigma_{1i}^2 + \sigma_{2i}^2) \]
Substituting $\sigma_i^2 = (\sigma_1^2 + \sigma_2^2)$, will result in:

$$V_i^4 - 2 \left[\frac{(P_i G_{ii} - Q_i B_{ii})}{Y_{ii}^2} + \frac{\sigma_i^2}{2 Y_{ii}^2}\right] V_i^2 + \frac{S_i^2}{Y_{ii}^2} = 0 \quad 3.21$$

Clearly, equation 3.21 is identical in form to the two-bus equation of 3.12 and the solution will have the form shown in equation 3.22.

$$V_i^2 = \frac{(P_i G_{ii} - Q_i B_{ii})}{Y_{ii}^2} + \frac{\sigma_i^2}{2 Y_{ii}^2} \pm \sqrt{\left[\frac{(P_i G_{ii} - Q_i B_{ii})}{Y_{ii}^2} + \frac{\sigma_i^2}{2 Y_{ii}^2}\right]^2 - \frac{S_i^2}{Y_{ii}^2}} \quad 3.22$$

At the maximum loading point, the second term is equal to zero and the maximum net power delivered to load bus $i$ is then calculated when,

$$V_i^2 = \frac{(P_i G_{ii} - Q_i B_{ii})}{Y_{ii}^2} + \frac{\sigma_i^2}{2 Y_{ii}^2} \quad 3.23$$

The solution for the voltage for $n$-bus system case is the same as that of two bus system, the only difference is the term $V_0^2$ which represents the equivalent voltage of one bus while in equation 3.23, $\frac{\sigma_i}{Y_{ii}}$ represents the aggregate voltage of power importing buses. To maintain the same behavior as for a two bus-system only the buses which are importing power to the bus of interest are taken into consideration while power exporting to the remaining buses are considered as a load. As shown in figure 3.3, bus $i$ represents the load bus of interest, and bus $k$, $l$, $m$ and $o$ are power importing buses and aggregated by a bus of voltage $\frac{\sigma_i}{Y_{ii}}$, while power exported to bus $j$ and $n$ represent the load for the bus of interest ($P_i$), in addition to any local load at the bus.
3.1.3 Determination of Weakest Bus

The first step in estimating the voltage collapse point for n-bus system using this method is to identify the weakest load bus in the power system. This work uses the P-index as trigger to give alarm when the system is approaching the point of collapse. When the P-index value of one of the load buses reaches 0.5, an alarm is raised to indicate voltage instability. This value was proposed in [14] and deemed worthy as an indicator. The critical bus is then determined using the difference between bus voltage and the corresponding axis voltage, the load bus with the minimum difference is the weakest bus in the system and to be used to calculate the point of collapse.

Figure 3.3

Power System Aggregation Based on Power Importing Buses
3.1.4 Application of the Voltage Instability Method to Test System

The multi-bus test system is the IEEE 14 bus system [15]. The P-index is evaluated for an increase in loading parameter $\lambda$ on all generator and load buses until it reaches 0.5. Based on the difference between the bus voltage and corresponding axis voltage for all the load buses in the system, bus 5 is determined as the weakest bus for this system. The axis voltage of bus 5 is shown in figure 3.3 and voltage $^2$ of the continuation curve of bus 5 is also plotted. It is clearly shown in the figure that the voltage collapse point is where the axis voltage intersects with the voltage $^2$ for the weakest bus in the system.

![Figure 3.4](image)

**Figure 3.4**

Bus-5 Voltage $^2$ and Axis Voltage

However, not all the cases of disturbances are solved using the voltage mode method. For example, if we test the outage of line 2-3 for the 14-bus system, the behavior of the weakest bus voltage with the axis voltage is shown in figure 3.5 and it is evident that the square of the voltage does not intersect with the axis voltage at the point of collapse. Therefore, the proposed method for the voltage mode fails for some cases, indicating that the collapse is not voltage triggered.
For these cases, a new approach is proposed in the next section to find the maximum loading point.

![Figure 3.5](image)

Bus-5 Voltage $^2$ and Axis Voltage for Case 2-3 Outage

### 3.1.5 Modification of Load-Flow for Voltage Mode Method

As mentioned above, the point of collapse is the point where the voltage of the weakest bus in the system intersects with its axis voltage term. To calculate the point of intersection, a change in load flow formulation is made and load parameter $\lambda$ is inserted into load flow equations.

For the standard load flow problem, injected powers can be written for the $i^{th}$ bus of an $n$-bus system as follows:

$$P_i = \sum_{k=1}^{n} |V_i||V_k|Y_{ik} \cos(\delta_i - \delta_k - \theta_{ik}) \quad 3.24$$

$$Q_i = \sum_{k=1}^{n} |V_i||V_k|Y_{ik} \sin(\delta_i - \delta_k - \theta_{ik}) \quad 3.25$$
\[ P_i = P_{Gi} - P_{Di}, \quad Q_i = Q_{Gi} - Q_{Di} \tag{3.26} \]

Where the subscripts G and D denote generation and load demand respectively on the related bus. In order to simulate a load change, a load parameter is inserted into demand powers \( P_{Di} \) and \( Q_{Di} \), mismatch equations for active and reactive power will be expressed as:

\[ \Delta P_i = -P_i + \lambda (P_{Gi} - P_{Di}), \quad \Delta Q_i = Q + \lambda (Q_{Gi} - Q_{Di}) \tag{3.27} \]

A new equation that describes the point of intersection is added, therefore, the load flow problem will calculate \( V \), \( \delta \) and loading parameter (\( \lambda \)) at the collapse point. The new equation is defined as the difference between the bus voltage and axis voltage for the weakest bus in the system, which has a value of zero at the collapse point and expressed as follows:

\[ F = V^2_p - (V^2_{axis0} + \text{slope} \times (\lambda - \lambda_0)) \tag{3.28} \]

where \( V^2_p \) represents the voltage of weakest bus in the system (P). The term \( V^2_{axis0} \) represents the initial axis voltage of \( V^2_p \), and is calculated for the weakest bus in the system when an alarm based on P-index is raised at a load parameter \( \lambda_0 \) (this is not \( \lambda \) at base load but rather at the point of alarm). The axis voltage is updated assuming its behavior is linear with the change in load parameter; this is shown to be largely true in Figs 3.4, 3.5. The slope of the axis voltage is calculated as follows:

Rewriting (3.22) as (with \( S_i^2 = \lambda^2 S_{i0}^2 \)):

\[ V_i^2 = V^2_{axis} \pm \sqrt{[V^2_{axis}]^2 - \frac{\lambda^2 S^2_{i0}}{Y_{ii}^2}} \tag{3.29} \]

Taking derivatives with respect to \( \lambda \):

\[ 2V_i \frac{dV_i}{d\lambda} = \frac{dV^2_{axis}}{d\lambda} \pm \sqrt{\frac{V^2_{axis} \frac{dV^2_{axis}}{d\lambda} - \lambda S^2_{i0}}{Y^2_{ii}}} \tag{3.30} \]
Manipulating,
\[
\frac{dV^2_{axis}}{d\lambda} = \text{slope} = \frac{2V_i \frac{dV_i}{d\lambda} + \lambda S^2_i}{\sqrt{\frac{[V^2_{axis}]^2}{Y_{ii}^2} - \frac{\lambda^2 S^2_i}{Y_{ii}^2}}} \frac{\sqrt{[V^2_{axis}]^2 - \frac{\lambda^2 S^2_i}{Y_{ii}^2}}}{1 + V^2_{axis}/\sqrt{[V^2_{axis}]^2 - \frac{\lambda^2 S^2_i}{Y_{ii}^2}}}
\]

The slope at \(\lambda_0\) is then calculated by substituting the terms \(\frac{dV_i}{d\lambda}|_{\lambda=\lambda_0}\) and other variables at this point.

The formulation of the Jacobian matrix for the modified load flow problem will change to include \(\lambda\) and it will be defined as follows:

\[
J = \begin{bmatrix}
\frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial V} & \frac{\partial P}{\partial \lambda} \\
\frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial V} & \frac{\partial Q}{\partial \lambda} \\
\frac{\partial F}{\partial \delta} & \frac{\partial F}{\partial V} & \frac{\partial F}{\partial \lambda}
\end{bmatrix}
\]

The load flow problem is of the form:

\[
\begin{bmatrix}
\Delta P_1 \\
\vdots \\
\Delta P_p \\
\Delta Q_1 \\
\vdots \\
\Delta Q_p \\
\Delta F
\end{bmatrix} = \begin{bmatrix}
\frac{\partial P_1}{\partial \delta_1} & \cdots & \frac{\partial P_1}{\partial \delta_p} & \frac{\partial P_1}{\partial V_1} & \cdots & \frac{\partial P_1}{\partial V_p} & \cdots & \frac{\partial P_1}{\partial \lambda} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
\frac{\partial P_p}{\partial \delta_1} & \cdots & \frac{\partial P_p}{\partial \delta_p} & \frac{\partial P_p}{\partial V_1} & \cdots & \frac{\partial P_p}{\partial V_p} & \cdots & \frac{\partial P_p}{\partial \lambda} \\
\frac{\partial Q_1}{\partial \delta_1} & \cdots & \frac{\partial Q_1}{\partial \delta_p} & \frac{\partial Q_1}{\partial V_1} & \cdots & \frac{\partial Q_1}{\partial V_p} & \cdots & \frac{\partial Q_1}{\partial \lambda} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
\frac{\partial Q_p}{\partial \delta_1} & \cdots & \frac{\partial Q_p}{\partial \delta_p} & \frac{\partial Q_p}{\partial V_1} & \cdots & \frac{\partial Q_p}{\partial V_p} & \cdots & \frac{\partial Q_p}{\partial \lambda} \\
\frac{\partial F}{\partial \delta_1} & \cdots & \frac{\partial F}{\partial \delta_p} & \frac{\partial F}{\partial V_1} & \cdots & \frac{\partial F}{\partial V_p} & \cdots & \frac{\partial F}{\partial \lambda}
\end{bmatrix} \begin{bmatrix}
\Delta \delta_1 \\
\vdots \\
\Delta \delta_p \\
\Delta V_1 \\
\vdots \\
\Delta V_p \\
\Delta \lambda
\end{bmatrix}
\]

*Bus \(p\) indicates the weakest bus in the system.

The new elements of the Jacobian matrix for bus \(i\) are defined as follows:
\[
\frac{\partial P_i}{\partial \lambda} = -(P_{Gi} - P_{Di}) \quad 3.34
\]
\[
\frac{\partial Q_i}{\partial \lambda} = -(Q_{Gi} - Q_{Di}) \quad 3.35
\]

F is a function of \( V_p \) and \( \lambda \) only. Derivatives with respect to angles and other voltages is zero. For the weakest bus in the system,
\[
\frac{\partial F}{\partial V_p} = -2V_p \quad 3.36
\]
\[
\frac{\partial F}{\partial \lambda} = \text{slope} \quad 3.37
\]

Using the modified load flow will result in the maximum loading parameter that satisfies the mismatch equation \((F = 0)\) which defines the collapse point.

### 3.2 Static Transfer Stability Limit Mode

For this mode, network collapse is not a load-bus voltage dependency problem. Instead, we are concerned with the generator angle behavior, and all the load buses will be eliminated using Kron reduction and system loads are converted into constant impedance equivalents for the purpose of this analysis.

#### 3.2.1 Kron Reduction

Prior to load bus elimination by Kron reduction, system loads are converted into constant impedance equivalents using equation 3.38,
\[
\tilde{Y}_{ii} = \tilde{Y}_{ii} - \frac{(P_{Li} + jQ_{Li})}{\tilde{V}_i^2} \quad 3.38
\]

where \( P_{Li} \) and \( Q_{Li} \) are injected active and reactive loads at the \( i^{th} \) bus.
Since the loads are modeled as passive admittances, all load nodes can be easily eliminated using the method of Kron’s reduction. The Kron’s reduction method is given as

\[ \bar{Y}_{jk(new)} = \bar{Y}_{jk} - \frac{(\bar{Y}_{jp} \bar{Y}_{pk})}{\bar{Y}_{pp}} \]

where “p” is the bus number of the bus to be eliminated, which is a load bus in our case. “j” is the row number and “k” is the column number.

### 3.2.2 Necessary Condition for Collapse

Now the system has only PV buses and a slack bus. For this type of analysis, we are interested in the power transfer between generation bus centers. Consider the equation of active power flow at the sending end of a line connecting buses j and k.

\[ P_{jk} = V_j^2 G_{jk} - V_j V_k Y_{jk} \cos(\delta_j - \delta_k + \theta_{jk}) \]

The likely mechanism for the voltage collapse at this mode is the static transfer stability limit (STSL) of the network branches, and since the lines connect PV buses with constant voltages, the system collapse is determined solely by the transmission-line angle limit \( \cos(\delta_j - \delta_k + \theta_{jk}) \). As a necessary condition first mentioned in [16], when a transfer of power takes place in a power system, at least one line must reach its static transfer stability limit (STSL) before the point of collapse is encountered.

In the proposed method, we are testing \( \cos(\delta_j - \delta_k + \theta_{jk}) \) for all the (reduced) branches in the system by simulating an increase in the loading parameter. As the load increases, the line flow changes correspondingly. The point of the first STSL occurs when one of the lines reaches its angle limit, so this point is tested as well as the next line with maximum STSL. In [16] it is
established that STSLs occur close to the collapse point, and thus the second point - if exists - should be even closer. It is necessary to carry out a Kron reduction at each step of the analysis, as the reduced network impedances will be voltage dependent.

3.2.3 Application of the STSL Method to Test System

Consider the case of line 2-3 outage for the IEEE-14 bus system where the voltage mode method failed to find the collapse accurately. The term \( \cos(\delta_j - \delta_k + \theta_{jk}) \) for all the branches in the system is plotted for an increase in loading parameter in figure 3.6. It is clearly shown in the figure that at least one branch (branch 3 \(-\) 1) has reached the cosine term limit (-1) before and close to the point of collapse. The graph shows that the second branch (branch 3 \(-\) 2) reaches the limit closer to the collapse point and thus more accurately estimates the maximum loading point.

![Figure 3.6](image)

Branches Static Transfer Stability Limit for Line 2-3 Outage
Redrawing the static transfer stability limit versus the loading parameter for the same system while considering the outage of line 13-14, results in a plot shown in figure 3.7. The search for the critical loading point could now be regarded as voltage mode problem since none of the branches has reached its static transfer stability limit before the point of collapse. The two cases of disturbance show fair confirmation of the theory.

![Figure 3.7](image)

**Figure 3.7**

Branches Static Transfer Stability Limit for Line 13-14 Outage

### 3.2.4 Modification of Load-Flow for STSL Mode Method

For this mode, the point of collapse is the point where the first or second branch reaches its limit of transmission angle (both are tested). Therefore, new equation is added to describe the cosine limit for the first or second branch. The new equation may be expressed as

\[
\Delta F = -1 \times \text{sign}\left(\cos(\delta_j - \delta_k - \theta_{redj_k})\right) + \cos(\delta_j - \delta_k - \theta_{redj_k})
\] 3.41
Where \( j \) and \( k \) are the two buses connecting the first or second branch (we are testing the two branches). \( \theta_{\text{red}}(jk) \) is branch angle for the reduced network. Since the cosine limit for the branch may be either 1 or -1, the sign of the cosine of the branch angle is used to define the limit in equation 3.41.

The new load flow problem is similar to the voltage mode as shown in equation 3.33; the difference is the new added equation. The new equation is a function of the angles that connect the first or second branch; therefore, the derivative with respect to the voltage and load parameter is zero.

\[
\frac{\partial F}{\partial V_i} = 0 \quad 3.42
\]

\[
\frac{\partial F}{\partial \lambda} = 0 \quad 3.43
\]

If \( j \) and \( k \) are the two buses that connect the first or second branch, the derivative of the new equation will be as follows:

\[
\frac{\partial F}{\partial \delta_j} = \sin(\delta_j - \delta_k - \theta_{\text{red}jk}) \quad 3.44
\]

\[
\frac{\partial F}{\partial \delta_k} = -\sin(\delta_j - \delta_k - \theta_{\text{red}jk})
\]

The derivative with respect to the other bus angles is zero.
CHAPTER 4
RESULTS AND DISCUSSION

4.1 Introduction

A MATLAB prototype of the proposed methods was written to identify the voltage collapse point using the two different approaches for IEEE 14, 39, 57, 118, and 300-bus systems. Furthermore, selected outages were performed, and the voltage and STSL mode methods were tested and compared for these outages. The following sections present a thorough illustration of the proposed approach performance and compare the results of the two modes to identify the collapse point.

4.2 Testing the Methods on the IEEE 14-Bus System

The two proposed methods were tested on all possible outages of the IEEE-14 bus system shown in Figure 4.1. The load multiplier was increased in steps and $\lambda_{max}$ was calculated for each mode. The voltage collapse point for the intact system was found to be at loading parameter $\lambda_{max}$ equal to 4.04 using the voltage limitation method and it took a total number of five iterations to arrive to this result. The STSL method, meanwhile, solved $\lambda_{max}$ of 3.99 in six iterations for this case. We will take the maximum of the two loading parameter as the solution so the voltage collapse for this case is voltage dependent problem. Table 4.1 presents the results obtained using the MATLAB prototype of the proposed voltage mode method along with those obtained using the STSL mode approach.
Based on table 4.1, the voltage mode method provided the largest $\lambda_{\text{max}}$ for vast majority of the cases, except for four cases where the STSL mode method provided results larger than voltage mode method such as line 2-3 outage case. By comparing the voltage and STSL mode, the maximum loading is the maximum of the two solutions providing a maximum error of 1.78% in the case of line 4-5 outage. There are some cases where the STSL method is unable to find solution for $\lambda_{\text{max}}$. 

Figure 4.1

Single Line Diagram of the IEEE 14-Bus System
Table 4.1

Comparison between the Voltage Mode and STSL Mode Methods for the IEEE 14-bus System

<table>
<thead>
<tr>
<th>Case</th>
<th>$\lambda_m$ (Actual)</th>
<th>$\lambda_m$ (Voltage Mode)</th>
<th>Weakest Bus</th>
<th>$\lambda_m$ (STSL Mode)</th>
<th>Weakest Branch</th>
<th>Maximum ($\lambda_{Voltage}, \lambda_{STSL}$)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intact</td>
<td>4.04</td>
<td>4.04</td>
<td>5</td>
<td>3.99</td>
<td>1-6</td>
<td>4.04</td>
<td>0.00%</td>
</tr>
<tr>
<td>1-2 Out</td>
<td>1.34</td>
<td>1.33</td>
<td>5</td>
<td>1.34</td>
<td>1-2</td>
<td>1.34</td>
<td>0.00%</td>
</tr>
<tr>
<td>1-5 Out</td>
<td>3.66</td>
<td>3.65</td>
<td>5</td>
<td>N.A</td>
<td>N.A</td>
<td>3.65</td>
<td>0.27%</td>
</tr>
<tr>
<td>2-3 Out</td>
<td>2.27</td>
<td>1.67</td>
<td>5</td>
<td>2.24</td>
<td>2-3</td>
<td>2.24</td>
<td>1.32%</td>
</tr>
<tr>
<td>2-4 Out</td>
<td>3.29</td>
<td>3.28</td>
<td>5</td>
<td>3.25</td>
<td>1-6</td>
<td>3.28</td>
<td>0.30%</td>
</tr>
<tr>
<td>2-5 Out</td>
<td>3.43</td>
<td>3.22</td>
<td>5</td>
<td>3.42</td>
<td>1-6</td>
<td>3.42</td>
<td>0.29%</td>
</tr>
<tr>
<td>3-4 Out</td>
<td>3.94</td>
<td>3.93</td>
<td>5</td>
<td>3.89</td>
<td>3-6</td>
<td>3.93</td>
<td>0.25%</td>
</tr>
<tr>
<td>4-5 Out</td>
<td>3.94</td>
<td>3.87</td>
<td>4</td>
<td>N.A</td>
<td>N.A</td>
<td>3.87</td>
<td>1.78%</td>
</tr>
<tr>
<td>4-7 Out</td>
<td>3.60</td>
<td>3.57</td>
<td>5</td>
<td>3.50</td>
<td>1-6</td>
<td>3.57</td>
<td>0.83%</td>
</tr>
<tr>
<td>4-9 Out</td>
<td>3.94</td>
<td>3.94</td>
<td>5</td>
<td>3.78</td>
<td>1-6</td>
<td>3.94</td>
<td>0.00%</td>
</tr>
<tr>
<td>5-6 Out</td>
<td>2.28</td>
<td>1.90</td>
<td>14</td>
<td>2.28</td>
<td>2-6</td>
<td>2.28</td>
<td>0.00%</td>
</tr>
<tr>
<td>6-11 Out</td>
<td>3.53</td>
<td>3.53</td>
<td>11</td>
<td>N.A</td>
<td>N.A</td>
<td>3.53</td>
<td>0.00%</td>
</tr>
<tr>
<td>6-12 Out</td>
<td>3.98</td>
<td>3.98</td>
<td>5</td>
<td>3.94</td>
<td>1-6</td>
<td>3.98</td>
<td>0.00%</td>
</tr>
<tr>
<td>6-13 Out</td>
<td>3.22</td>
<td>3.21</td>
<td>13</td>
<td>N.A</td>
<td>N.A</td>
<td>3.21</td>
<td>0.31%</td>
</tr>
<tr>
<td>7-9 Out</td>
<td>2.88</td>
<td>2.85</td>
<td>9</td>
<td>N.A</td>
<td>N.A</td>
<td>2.85</td>
<td>1.04%</td>
</tr>
<tr>
<td>9-10 Out</td>
<td>4.01</td>
<td>4.01</td>
<td>5</td>
<td>3.95</td>
<td>1-6</td>
<td>4.01</td>
<td>0.00%</td>
</tr>
<tr>
<td>9-14 Out</td>
<td>3.70</td>
<td>3.69</td>
<td>14</td>
<td>N.A</td>
<td>N.A</td>
<td>3.69</td>
<td>0.27%</td>
</tr>
<tr>
<td>10-11 Out</td>
<td>3.74</td>
<td>3.73</td>
<td>10</td>
<td>N.A</td>
<td>N.A</td>
<td>3.73</td>
<td>0.27%</td>
</tr>
<tr>
<td>12-13 Out</td>
<td>4.03</td>
<td>4.03</td>
<td>5</td>
<td>3.98</td>
<td>1-6</td>
<td>4.03</td>
<td>0.00%</td>
</tr>
<tr>
<td>13-14 Out</td>
<td>3.25</td>
<td>3.25</td>
<td>14</td>
<td>N.A</td>
<td>N.A</td>
<td>3.25</td>
<td>0.00%</td>
</tr>
</tbody>
</table>

*N.A: Not Applicable for this case.

4.3 Testing the methods on the IEEE 39-bus system

When applied to the IEEE-39 bus system shown in Figure 4.2, the STSL mode scheme took five iterations to identify the collapse point for the intact system the intact system and it was found to be 2.16, while the voltage mode scheme resulted $\lambda_{\text{max}}$ of 2.04. In this case, the voltage collapse for the intact system is solved using the STSL mode method with error of 0.46%.

Using the two proposed methods to find the collapse point provided maximum error of 2.99% in case of line 10-13 outage where the problem is voltage limitation driven problem.
Summary of using the two proposed methods to identify the voltage collapse point on different outages scenarios is shown in Table 4.2.

Figure 4.2

Single Line Diagram of the IEEE 39-Bus System

Table 4.2

Comparison between the Voltage Mode and STSL Mode Methods for the IEEE 39-bus System

<table>
<thead>
<tr>
<th>Case</th>
<th>$\lambda_m$ (Actual)</th>
<th>$\lambda_m$ (Voltage Mode)</th>
<th>Weakest Bus</th>
<th>$\lambda_m$ (STSL Mode)</th>
<th>Weakest Branch</th>
<th>Maximum ($\lambda_{\text{Voltage}}, \lambda_{\text{STSL}}$)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intact</td>
<td>2.17</td>
<td>2.04</td>
<td>10</td>
<td>2.16</td>
<td>35-39</td>
<td>2.16</td>
<td>0.46%</td>
</tr>
<tr>
<td>3-18 Out</td>
<td>2.16</td>
<td>N.A</td>
<td>N.A</td>
<td>2.12</td>
<td>35-39</td>
<td>2.12</td>
<td>1.53%</td>
</tr>
<tr>
<td>6-7 Out</td>
<td>1.94</td>
<td>1.91</td>
<td>7</td>
<td>1.89</td>
<td>35-39</td>
<td>1.91</td>
<td>1.55%</td>
</tr>
<tr>
<td>10-13 Out</td>
<td>2.01</td>
<td>1.95</td>
<td>12</td>
<td>N.A</td>
<td>N.A</td>
<td>1.95</td>
<td>2.99%</td>
</tr>
<tr>
<td>15-16 Out</td>
<td>1.80</td>
<td>1.79</td>
<td>15</td>
<td>N.A</td>
<td>N.A</td>
<td>1.79</td>
<td>0.55%</td>
</tr>
<tr>
<td>17-18 Out</td>
<td>2.11</td>
<td>2.06</td>
<td>7</td>
<td>2.05</td>
<td>35-39</td>
<td>2.06</td>
<td>2.37%</td>
</tr>
<tr>
<td>17-27 Out</td>
<td>2.16</td>
<td>N.A</td>
<td>N.A</td>
<td>2.13</td>
<td>35-39</td>
<td>2.13</td>
<td>1.39%</td>
</tr>
<tr>
<td>25-26 Out</td>
<td>2.15</td>
<td>N.A</td>
<td>N.A</td>
<td>2.09</td>
<td>36-39</td>
<td>2.09</td>
<td>2.79%</td>
</tr>
</tbody>
</table>

*N.A: Not Applicable for this case.
4.4 Testing the methods on the IEEE 57-bus system

In this case, the resulting loading parameter $\lambda_{\text{max}}$ for the intact IEEE-57 bus system shown in Figure 4.3 was found to be 2.06 using the voltage mode method while no solution was found using the STSL mode method for this case and for the other outage cases. The two proposed methods were used to identify the voltage collapse point on different outages scenarios and Table 4.3 summarizes the results obtained from the simulation.

![Single Line Diagram of the IEEE 57-Bus System](image)

Figure 4.3

Single Line Diagram of the IEEE 57-Bus System
Table 4.3

Comparison between the Voltage Mode and STSL Mode Methods for the IEEE 57-bus System

<table>
<thead>
<tr>
<th>Case</th>
<th>$\lambda_m$ (Actual)</th>
<th>$\lambda_m$ (Voltage Mode)</th>
<th>Weakest Bus</th>
<th>$\lambda_m$ (STSL Mode)</th>
<th>Weakest Branch</th>
<th>Maximum ($\lambda_{Voltage}, \lambda_{STSL}$)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intact</td>
<td>2.07</td>
<td>2.06</td>
<td>31</td>
<td>N.A</td>
<td>N.A</td>
<td>2.06</td>
<td>0.48%</td>
</tr>
<tr>
<td>4-5 Out</td>
<td>2.06</td>
<td>2.06</td>
<td>31</td>
<td>N.A</td>
<td>N.A</td>
<td>2.06</td>
<td>0.00%</td>
</tr>
<tr>
<td>1-15 Out</td>
<td>2.02</td>
<td>2.01</td>
<td>31</td>
<td>N.A</td>
<td>N.A</td>
<td>2.01</td>
<td>0.49%</td>
</tr>
<tr>
<td>13-29 Out</td>
<td>2.03</td>
<td>2.03</td>
<td>31</td>
<td>N.A</td>
<td>N.A</td>
<td>2.03</td>
<td>0.00%</td>
</tr>
<tr>
<td>7-29 Out</td>
<td>1.23</td>
<td>1.22</td>
<td>29</td>
<td>N.A</td>
<td>N.A</td>
<td>1.22</td>
<td>0.81%</td>
</tr>
<tr>
<td>23-24 Out</td>
<td>1.94</td>
<td>1.93</td>
<td>31</td>
<td>N.A</td>
<td>N.A</td>
<td>1.93</td>
<td>0.51%</td>
</tr>
<tr>
<td>24-25 Out</td>
<td>1.83</td>
<td>1.80</td>
<td>31</td>
<td>N.A</td>
<td>N.A</td>
<td>1.80</td>
<td>1.64%</td>
</tr>
<tr>
<td>24-26 Out</td>
<td>1.84</td>
<td>1.84</td>
<td>31</td>
<td>N.A</td>
<td>N.A</td>
<td>1.84</td>
<td>0.00%</td>
</tr>
<tr>
<td>22-38 Out</td>
<td>1.88</td>
<td>1.88</td>
<td>31</td>
<td>N.A</td>
<td>N.A</td>
<td>1.88</td>
<td>0.00%</td>
</tr>
<tr>
<td>37-38 Out</td>
<td>1.24</td>
<td>1.23</td>
<td>34</td>
<td>N.A</td>
<td>N.A</td>
<td>1.23</td>
<td>0.81%</td>
</tr>
</tbody>
</table>

*N.A: Not Applicable for this case.

Based on table 4.3 it is evident that all of the IEEE-57 bus system selected outages were solved using voltage mode method with a maximum error of 1.64%.

4.5 Testing the methods on the IEEE 118-bus system

The two proposed methods were used to identify the voltage collapse point on different outages scenarios for the IEEE-118 bus system shown in Figure 4.4. The loading parameter $\lambda_{max}$ was found to be 3.04 and 2.87 for the voltage and STSL methods respectively and the maximum of the two (3.04) resulted an error of 5.00%. Table 4.4 summarizes the results obtained from the simulation for different scenarios.
Figure 4.4

Single Line Diagram of the IEEE 118-Bus System

Table 4.4

Comparison between the Voltage Mode and STSL Mode Methods for the IEEE 118-bus System

<table>
<thead>
<tr>
<th>Case</th>
<th>$\lambda_m$ (Actual)</th>
<th>$\lambda_m$ (Voltage Mode)</th>
<th>Weakest Bus</th>
<th>$\lambda_m$ (STSL Mode)</th>
<th>Weakest Branch</th>
<th>Maximum $(\lambda_{Voltage}, \lambda_{STSL})$</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intact</td>
<td>3.20</td>
<td>3.04</td>
<td>38</td>
<td>2.87</td>
<td>62-70</td>
<td>3.04</td>
<td>5.00%</td>
</tr>
<tr>
<td>23-24 Out</td>
<td>2.98</td>
<td>2.72</td>
<td>38</td>
<td>2.80</td>
<td>55-112</td>
<td>2.80</td>
<td>6.04%</td>
</tr>
<tr>
<td>26-30 Out</td>
<td>2.61</td>
<td>2.52</td>
<td>38</td>
<td>2.30</td>
<td>72-76</td>
<td>2.52</td>
<td>3.45%</td>
</tr>
<tr>
<td>49-69 Out</td>
<td>3.19</td>
<td>3.09</td>
<td>47</td>
<td>3.02</td>
<td>72-105</td>
<td>3.09</td>
<td>3.13%</td>
</tr>
<tr>
<td>68-69 Out</td>
<td>2.75</td>
<td>2.39</td>
<td>47</td>
<td>2.66</td>
<td>10-80</td>
<td>2.66</td>
<td>3.27%</td>
</tr>
</tbody>
</table>
Based on table 4.4, voltage collapse for the IEEE-118 bus system is voltage dependent problem for some cases and STSL dependent problem for other cases and the maximum error is 6.04\% in case of line 23-24 outage.

4.6 Testing the methods on the IEEE 300-bus system

Similar to the previous test systems, different outages were performed on the IEEE-300 bus system shown in figure 4.5 and the performance of the two proposed methods was examined. However while the voltage mode method solved the intact system and resulted a loading limit of 1.42 with of 0.70\%, error, the STSL mode method failed to find the loading limit for this case. Table 4.5 summarizes the results obtained from the MATLAB script.

Figure 4.5
Single Line Diagram of the IEEE 300-Bus System
Table 4.5

Comparison between the Voltage Mode and STSL Mode Methods for the IEEE 300-bus System

<table>
<thead>
<tr>
<th>Case</th>
<th>$\lambda_m$ (Actual)</th>
<th>$\lambda_m$ (Voltage Mode)</th>
<th>Strongest Bus</th>
<th>$\lambda_m$ (STSL Mode)</th>
<th>Strongest Branch</th>
<th>Maximum $(\lambda_{\text{Voltage}}, \lambda_{\text{STSL}})$</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intact</td>
<td>1.43</td>
<td>1.42</td>
<td>171</td>
<td>N.A</td>
<td>N.A</td>
<td>1.42</td>
<td>0.70%</td>
</tr>
<tr>
<td>3-4 Out</td>
<td>1.43</td>
<td>1.42</td>
<td>171</td>
<td>N.A</td>
<td>N.A</td>
<td>1.42</td>
<td>0.70%</td>
</tr>
<tr>
<td>40-68 Out</td>
<td>1.35</td>
<td>N.A</td>
<td>N.A</td>
<td>1.21</td>
<td>212-265</td>
<td>1.21</td>
<td>10.37%</td>
</tr>
<tr>
<td>116-119 Out</td>
<td>1.42</td>
<td>1.42</td>
<td>171</td>
<td>N.A</td>
<td>N.A</td>
<td>1.42</td>
<td>0.00%</td>
</tr>
</tbody>
</table>

*N.A: Not Applicable for this case.

The IEEE 300-bus system is heavily stressed with a small margin of loadability. This makes the percentage error in estimation of the loading limit very sensitive to load discrepancies. Therefore, as shown in the table above there is a case where STSL mode method estimated the voltage collapse point with percentage error of 10.37%.
CHAPTER 5
CONCLUSION AND FUTURE WORK

5.1 Conclusion

In this study, new algorithm was developed for voltage collapse point estimation and this algorithm proposed two new methods. The first method took into consideration the voltage instability of the load buses based on the basic power flow equations and it has a very simple structure and can be handled easily. The other method is based on the angle instability of the generator buses and the objective of this method is to search for the maximum loading point of the power system when the collapse happens due to static transfer stability limit for PV system. Both methods are used for each system and the maximum net power delivered is the maximum of the two.

The performance of the new methods was investigated on a variety of test systems namely, the IEEE 14, IEEE 39, IEEE 57, IEEE 118, and IEEE 300-bus system. It was evident that the proposed algorithm to an acceptable degree succeeded in estimating the maximum loading points of these systems for all the cases either using voltage mode or static transfer stability limit mode.

The distinction of the voltage and angle limitation behaviors at the point of collapse is a major contribution of this work, and incorporation of these distinctions into power flows is what makes the suggested algorithm powerful when compared to the other methods in the literature. The estimated limit is always close to exact or conservative since it represents an actual solution.
All the proposed methods and indices in the literature are based on the fact that the system experiences collapse due to voltage instability of the load buses. It is therefore hoped that this effort opens the door to a new approach for voltage stability based on the distinction between voltage and angle modes of collapse.

5.2 Future Work

Both proposed voltage and static transfer stability limit formulations need further investigations to improve accuracy. The weakness of the STSL method is that more than one branch may need to be investigated to estimate the point of collapse with reasonable accuracy. Additionally, a Kron reduction is required at each iteration, which adds time to the loadflow, particularly for large systems. It is recommended to continue research to find a transfer based singularity based on a system aggregation or through Jacobian matrix examination rather than a branch-by-branch investigation.
REFERENCES


VITA

Areeg Ahmed was born in Khartoum, Sudan in 1993. Before attending The University of Tennessee at Chattanooga as a Master of Science candidate, she attended the University of Khartoum in Khartoum, Sudan, where she earned her B.sc - Bachelor of Science in Electrical Engineering, with First Class, in 2015. In August 2017, she was awarded a graduate assistantship at The University of Tennessee at Chattanooga. Areeg was awarded a Master of Science degree in Electrical Engineering in August 2019. Her interests in power system include power system analysis, power system stability and control.

Currently, Areeg is working at Black & Veatch in Chattanooga, Tennessee, as an electrical engineer.